On the All Occurrences of a Word in a Text

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Abstract. In this paper a simple straight string search algorithm is presented. For a string $s$ that consists of $n$ characters and a pattern $p$ that consists of $m$ characters the order of comparisons is $O(n \cdot m)$, $0 < m \leq n$, in the worst case, but the average time complexity is good. The algorithm presented finds all occurrences of $p$ in $s$. It do not use a precompiling of the pattern $p$.

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Key words: direct, string, pattern, search

1 Introduction

The string matching problem is following. Given an array $s[0..n-1]$ of $n$ characters and an array $p[0..m-1]$ of $m$ characters where $0 < m \leq n$, the task is to find all occurrences of $p$ in $s$. The string $s$ is regarded as a text and the string $p$ as a word(pattern). Generally, $s$ and $p$ are item.

In [W86] it is presented a direct method to determine the first occurrence of $p$ in $s$. In the same book it is presented the fact that the algorithm proposed is very inefficient, for example, if the pattern is $p=a^{n-1}b$ and the string is $s=a^{n-1}b$, then $m \cdot n$ comparisons are necessary to determine that $p$ is in $s$.

In this direct method the pattern and the text are aligned at the left ends. The searching begins with $p_0$ and $s_0$. If a mismatch appears then a new searching begins always with $p_0$, the first character of the pattern.

2 The algorithm

The algorithm proposed by us begins with $p$ and $s$ aligned at the left ends too but in the case that a mismatch occurs in the process of comparisons of $p$ and $s$ ($p_j \neq s_j$) then the searching continues with the character of $p$ which produced the mismatch, that is $p_j$, which is searched between $s_{j+1}$ and $s_{n-m+j}$. On this idea the algorithm is built. It will contain the followings.

1. One compares successively $p_0$ with $s_i$, $i=0,1,\ldots,n-m$. If it exists no match of the $p_0$ with $s_i$, $i=0,1,\ldots,n-m$ then ' $p$ is not in $s$' and the process is terminated.

2. If $s_i$ is the first match of $p_0$ then one compares successively $p_1$ with $s_{i+1}$, $p_2$ with $s_{i+2}$ etc. If all $p_j$ match with $s_{i+j}$, $j=0,1,\ldots,m-1$ then this is the first occurrence of $p$ in $s$. A new searching is resumed beginning with $p_0$ and $s_{i+m}$.
3. If in the process of searching a mismatch occurs between \( p_j \) and \( s_{i+j} \) \((p_j \neq s_{i+j})\) then \( p_j \) is searched in the rest of string \( s \) between \( s_{i+j+1} \) and \( s_{n-m+j} \). If \( p_j \) is not in this rest then the searching is ended.

4. If in the substring \( s_{i+j+1}, ..., s_{n-m+j} \) there exists a character which match with \( p_j \), one renames this character \( s_i \). Therefore \( p_j = s_i \). In this case one compares the left and right neighbours of \( p_j \) and \( s_i \) that is \( p_0, p_1, ..., p_j, ..., p_{m-1} \) with corresponding \( s_{i-j}, ..., s_i, ..., s_{i-j+m-1} \). If all occur then this is an occurrence of \( p \) in \( s \) and the process of searching is resumed. If in the time of verification the neighbours of \( p_j \) and \( s_i \) a mismatch occurs then a new searching of \( p_j \) begins with the character \( s_{i+1} \).

5. The algorithm stops if \( i >= n - m + j \).

**Example.**

\[ p = abcd \quad (m=4) \]
\[ s = xabcdxabxxaycdabcd \quad (n=19) \]

In this example there are 23 comparisons to find two occurrences of \( p \) in \( s \).

The complete algorithm, presented as a procedure named DO3 (written in a Pascal-like language described in [HS83]), is the following.

```plaintext
procedure DO3(s,p,n,m)
//find all occurrences of the word p(0:m-1)//
//in the string s(0:n-1) if this exists. If yes//
//then procedure writes 'p is in s' else it//
//write 'p is not in s'. 0<m<=n//
char p(0:m-1),s(0:n-1); integer i,j,m,n,k; boolean f;
i:=0; f:=false;
loop
  j:=0;
  while (j<m) and (p(j)=s(i)) do i:=i+1; j:=j+1 repeat;
  if (j=m) then write('p is in s');f:=true;cycle endif
  // the character p(j) is a mismatch:p(j)!=s(j) //
  i:=i+1;
  while (i<=n-m+j) and (p(j)!=s(i)) do i:=i+1 repeat
```

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if \(i>n-m+j\) and not f then exit endif;
// it exists i thus \(p(j)=s(i)\), one verifies the //
// left and right neighbours of \(p(j)\) and \(s(i)\)

\[k:=0;\]
while (\(k<=m-1\)) and \((p(k)=s(i-j+k)\)) do \(k:=k+1\) repeat;
if \(k=m\) then write(‘p is in \(s’\)); \(f:=\text{true}\); \(i:=i-j+m\)
else goto 1 endif
until \(i>n-m+j\) repeat;
if not f then write(‘p is not in \(s’\)) endif

endDO3;

3 Number of comparisons

The maximum number of comparisons to determine that ‘p’ is or it is not in \(s\), theoretically, it is obtained when, after \(p_k=s_k\), \(k=0,1,...,j-1\) match, it appears \(p_j \neq s_j\), but \(p_j = s_i\), \(i = j+1,...,n - m + j\) and all the left neighbours of \(p_j\) match with the corresponding neighbours of \(s_i\), and the right neighbours of \(p_j\), that is, \(p_{j+1}, p_{j+2},..., p_{m-2}\) match with the right corresponding neighbours of \(s_i\) excepting \(p_{m-1}\). For \(i = n - m + j\), \(p_{m-1}\) may or it may not match with his corresponding in \(s\). Therefore for:

\[i = j+1, \quad p_0 = s_1,..., p_j = s_i,..., p_{m-2} = s_{m-1}; \quad p_{m-1} \neq s_m \text{ there are } m \text{ comparisons;}
\]
\[i = j+2, \quad p_0 = s_2,..., p_j = s_i,..., p_{m-2} = s_{m}; \quad p_{m-1} \neq s_{m+1} \text{ there are } m \text{ comparisons;}
\]

\[i = n - m + j, \quad p_0 = s_{n-m+j},..., p_j = s_i,..., p_{m-2} = s_{n-2} \quad \text{and} \quad p_{m-1} = s_{n-1} \text{ or} \quad p_{m-1} \neq s_{n-1}, \text{ there are } m \text{ comparisons.} \]

Therefore in all it exists \(j+1\) comparisons \(p_k\) with \(s_k, k = 0,1,...,j;\) between \(j+1\) and \(n - m + j\) there exists \((n - m + j)-(j+1)+1 = n-m\) cases for which \(p_j\) may match with \(s_i, i = j+1, j+2,...,n - m + j\) and the neighbours of \(p_j\), that is \(p_0, p_1,..., p_{m-2}\) match with the corresponding neighbours of \(s_i\), but \(p_{m-1} \neq s_{m+k}, k = -1, 0, 1,..., n - m - 1\). Possibly, \(p_{m-1} = s_{n-1}\). Every case gives \(m\) comparisons. Hence the maximum number of comparisons is

\[N_{\text{max}} = j + 1 + (n - m) * m \leq m - 1 + 1 + (n - m)m = m(n - m + 1).\]

The complexity of the algorithm DO3 is \(O(n.m)\) too.

But in the most unfavourable cases the algorithm DO3 reduces the maximum number of comparisons from \(m * n\) as in algorithm presented by N.Wirth in [W86] to \(m(n - m + 1)\).

For the example \(p=a^{m-1}b\) and \(s=a^{n-1}b\) presented in Section 1, the algorithm DO3 carries out \(n + m - 1\) comparisons.

4 Profiling

The variant of this algorithm(OD) written to determine the first occurrence of \(p\) in \(s\) [D98] has been compared with a direct method(DIR) presented in [W86] and the Boyer-Moore algorithm(BM) [BM77]. The tests have been realized for different
values of \( p(m=5, 10, 20, 50, 100) \) and \( s(n=1000, 2000, 3000, 4000, 5000) \). The \( p \) and \( s \) have been generated randomly. One generated sequences of \( m \) and \( n \) decimal integer random numbers between 32-127 and one has taken the ASCII corresponding characters for \( p \) respectively for \( s \). For the same \( m \) and \( n \) the three methods have been executed 10 times. The average time for an \( m \) and five values for \( n(=1000, 2000, 3000, 4000, 5000) \) are written down in the following table

<table>
<thead>
<tr>
<th>( m )</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>OD</td>
<td>0.52</td>
<td>0.20</td>
<td>0.56</td>
<td>0.24</td>
<td>0.30</td>
<td>0.364</td>
</tr>
<tr>
<td>DIR</td>
<td>0.46</td>
<td>0.58</td>
<td>0.24</td>
<td>0.34</td>
<td>0.58</td>
<td>0.432</td>
</tr>
<tr>
<td>BM</td>
<td>0.12</td>
<td>0.22</td>
<td>0.44</td>
<td>0.42</td>
<td>0.22</td>
<td>0.284</td>
</tr>
</tbody>
</table>

Between the average times of three methods there are the relations

\[ t_{OD} = 1.28t_{BM}; \quad t_{DIR} = 1.18t_{OD}. \]

But if the three methods are executed 100 times then the values are the following

<table>
<thead>
<tr>
<th>( m )</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>OD</td>
<td>0.362</td>
<td>0.374</td>
<td>0.396</td>
<td>0.376</td>
<td>0.368</td>
<td>0.372</td>
</tr>
<tr>
<td>DIR</td>
<td>0.408</td>
<td>0.398</td>
<td>0.408</td>
<td>0.414</td>
<td>0.396</td>
<td>0.404</td>
</tr>
<tr>
<td>BM</td>
<td>0.364</td>
<td>0.350</td>
<td>0.308</td>
<td>0.300</td>
<td>0.312</td>
<td>0.326</td>
</tr>
</tbody>
</table>

In this case the relations are

\[ t_{OD} = 1.14t_{BM}; \quad t_{DIR} = 1.08t_{OD}. \]

5 Correctness of the algorithm

**Theorem.** The algorithm DO3 works correctly.

**Proof.** To proof the correctness of the algorithm we use a proof table [TBCG92]

procdure DO3(s,p,n,m)
char p(0:m-1),s(0:n-1); integer i,j,m,n,k; boolean f;
{pre:input=(p0,p1,...,pm-1)∧(s0,s1,...,sn-1)∧
n ≥ m > 0∧∀i∈{0,1,...,n-1}:si are characters∧
∀j∈{0,1,...,m-1}:pj are characters}
f:=false; i:=0;
loop
j:=0;
while \((j<m)\) and \((p(j)=s(i))\) do
\{inv:\forall h \in \{0,1,\ldots,j-1\}: p_h = s_h \land 0 \leq j, i \leq m\}
i:=i+1; j:=j+1 repeat;
\{\forall h \in \{0,1,\ldots,j-1\}: p_h = s_h \land (j=m \lor p_j \neq s_i)\}
if \((j=m)\) then write('p is in s'); f:=true;
\{output f=true\}
cycle endif
// the character \(p(j)\) is a mismatch: \(p(j) \neq s(j)\) //
{f=false \land j<m \land p_j \neq s_i}\n1:i:=i+1;
\{0<i \leq n-m+j \land p_j \neq s_i\} \land \forall i>n-m+j\}
while \((i=n-m+j)\) and \((p(j) \neq s(i))\) do
\{inv: \(p_j \neq s_{i-1} \land i \leq n-m+j\}\n1:i:=i+1 repeat;
\{(p_j \neq s_{i-1} \land -(i=n-m+j \land p_j \neq s_i)) \equiv \ (p_j \neq s_{i-1} \land i>n-m+j) \lor (s_i = p_j \land i=n-m+j)\}\nif \(i>n-m+j\) and not \(f\) then
\{p_j \neq s_i\}
exit endif;
\{p_j = s_i \land i \leq n-m+j\}
// it exists \(i\) thus \(p(j)=s(i)\) //
// one verifies the left and right neighbours of \(p(j)\) and \(s(i)\) //
k:=0;
while \((k=m-1)\) and \((p(k)=s(i+j+k))\) do
\{inv:\forall h \in \{0,1,\ldots,k-1\}: p_h = s_{i+j+k} \land 0 \leq k \leq m\}
k:=k+1 repeat;
\{(\forall h \in \{0,1,\ldots,k-1\}: p_h = s_{i+j+k} \land -(k=n-m \land p_k = s_{i+j+k})\} \equiv \ (\forall k \in \{0,1,\ldots,m-1\}: p_k = s_{i+j+k} \land k=m) \land \ (\forall h \in \{0,1,\ldots,k-1\}: p_h = s_{i+j+k} \land p_k \neq s_{i+j+k})\}
if \(k=m\) then write('p is in s'); f:=true; i:=i+j+m
\{p_k = s_{i+j+k}\}
else
\{\exists k \in \{0,1,\ldots,m-1\}: p_k \neq s_{i+j+k}\}
goto 1
endif
until \(i=n-m+j\) repeat;
\{f=false \lor f=true \land 0 \leq j=n \land i<n-m+j\}
if not \(f\) then write('p is not in s\') endif;
\{post: output = \}\end DO3;

The justifications are based on the application of logical equivalences and the rules of inference to the sequence of Pascal statements. These are:
i) the assignment rule of inference
\{P(e)\} \ v:=e \ {P(v)}

ii) the conditional rules of inference
a) \{P \land B\} \ s \ {Q}\n
b) \{P \land \neg B\} \ s1 \ {Q}\n
55
\[
P \land \neg B \Rightarrow Q
\]

\[
\{P\} \text{ if } B \text{ then } s \{Q\}
\]

\[
P \land \neg B \} s2 \{Q\}
\]

\[
\{P\} \text{ if } B \text{ then } s1 \text{ else } s2 \{Q\}
\]

iii) the loop rules of inference

a) \{inv \land B\} \text{ s } \{inv\}

b) \{inv \land B\} \text{ s } \{inv\}

\{inv\} \text{ while } B \text{ do } s \{\text{inv} \land \neg B\}

\{inv\} \text{ repeat } s \text{ until } B \{\text{inv} \land B\}

where \(P, Q\) denote propositions, \(B\)-Boolean expression, \(\text{inv}\)-the invariant of the loop and \(s, s1, s2\) are statements.

Conclusions

1) Algorithm OD is faster than algorithm DIR in average time;
2) There are pairs of \(p\) and \(s\) where algorithms OD or DIR are faster than algorithm BM;
3) At limit, the average times of the three methods tend to approach;
4) Possibly, for other \(p\) and \(s\), the relations between the average times of the three methods can be slight different.

References


[D93] O.Dogaru, Algorithm of straight string search, Proceedings of the 9th Romanian SYmposium on Computer Science (ROSYCS), University of Iasi,(1993), pp.172-177


