Two-dimensional bitwise memory matrix: A tool for optimal parallel approximate pattern matching

Jan Šupol and Bořivoj Melichar

`jan.supol@gmail.com, melichar@fel.cvut.cz`

Department of Computer Science & Engineering
Faculty of Electrical Engineering
Czech Technical University in Prague
Karlovo nám. 13, 121 35 Prague 2

Contents

- Two dimensional memory matrix
- Parallel pattern matching
- $\mathcal{O}(1)$ time for the exact pattern matching
- $\mathcal{O}(k)$ for the approximate pattern matching
- $\mathcal{O}(n)$ processors
- *CREW PRAM*
- Based on bit-parallel *NFA* simulation

Prior algorithms

- Zvi Galil. A constant-time optimal parallel string-matching algorithm. *Proceedings of the twenty-fourth annual ACM symposium on Theory of Computing*, 69–76, 1992.
- $\mathcal{O}(1)$ time
- $\mathcal{O}(n\ m)$ processors
- *CRCW PRAM*

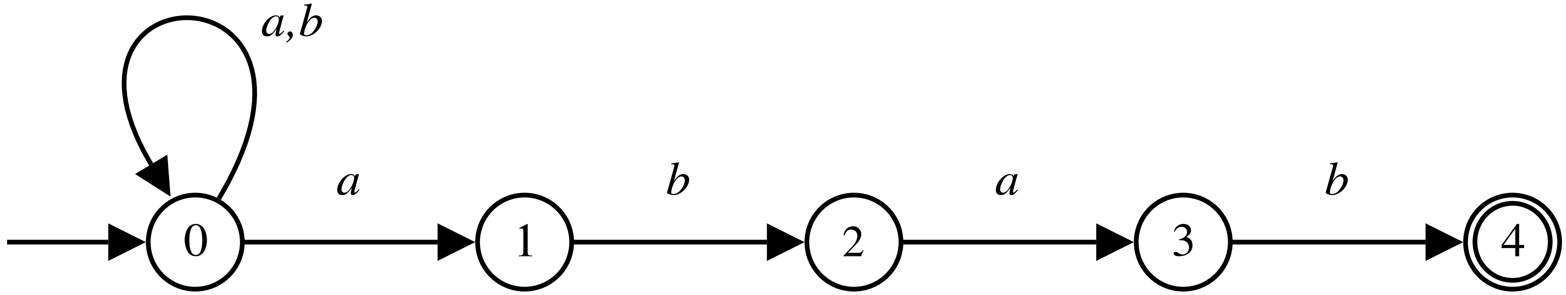
Prior algorithms

- G. M. Landau and U. Vishkin. Fast parallel and serial approximate string matching. *Journal of Algorithms*, 10(2):157–169, 1989.
- $\mathcal{O}(\mathit{log}\, m + k)$ time
- $\mathcal{O}(n)$ processors
- *CRCW PRAM*

Prior algorithms

- Y. Jiang and A. H. Wright. $\mathcal{O}(k)$ Parallel Algorithms for Approximate String Matching. *Neural, Parallel and Scientific Computations*,1:443–452, 1993.
- $\mathcal{O}(k)$ time
- $\mathcal{O}(n\ m)$ processors
- Priority *CRCW PRAM*

Bit-parallel algorithm



$$\vec{r} = r_0 r_1 r_2 r_3 r_4$$

$shl(\vec{r}) \land$ *remove inactive states.*

“Shift-or” bit-parallel algorithm

- Matrix $R^l, 0 \leq l \leq k$ of size $m \times (n+1)$
- Matrix D of size $m \times |\mathcal{A}|$
  Example: table D for $P = adbbca$

D	a	b	c	d	$\mathcal{A} \setminus \{a,b,c,d\}$
a	0	1	1	1	1
d	1	1	1	0	1
b	1	0	1	1	1
b	1	0	1	1	1
c	1	1	0	1	1
a	0	1	1	1	1

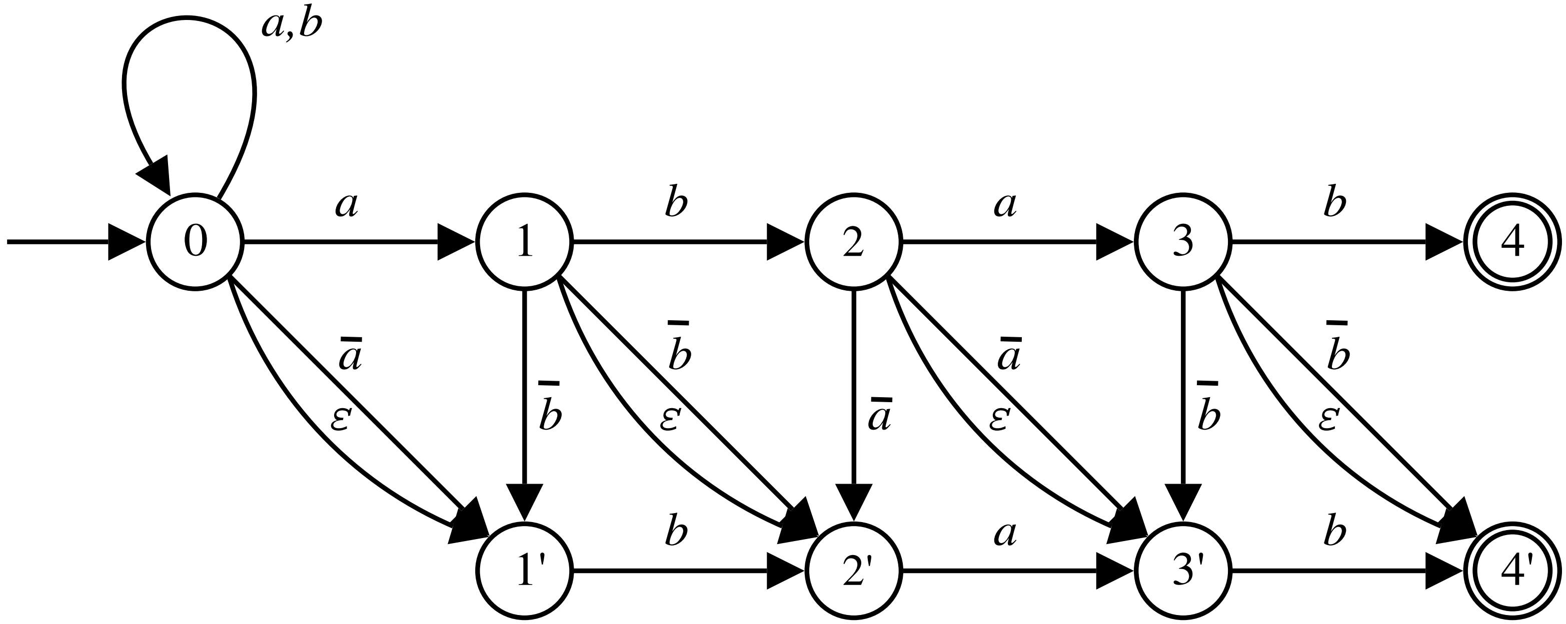
- k is the maximum number of edit operations in pattern P

“Shift-or” bit-parallel algorithm

Exact pattern matching

$$\begin{array}{rcll} r^0_{j,0} & = & 1, & 0 < j \leq m \\ R^0_i & = & \mathtt{shl}(R^0_{i-1})\ \mathtt{or}\ D[t_i], & 0 < i \leq n \end{array}$$

Bit-parallel algorithm



$$\begin{aligned} R_i^l \;=\; & (\mathtt{shl}(R_{i-1}^l)\ \mathtt{or}\ D[t_i]) \\ & \mathtt{and}\ \mathtt{shl}({\color{green}R_{i-1}^{l-1}}\ \mathtt{and}\ {\color{orange}R_i^{l-1}}) \\ & \mathtt{and}\ ({\color{blue}R_{i-1}^{l-1}\ \mathtt{or}\ V}), \qquad 0 < i \le n, 0 < l \le k \end{aligned}$$

"Shift-or" bit-parallel algorithm

Approximate pattern matching

$$
\begin{aligned}
r^l_{j,0} &= 0, && 0 < j \leq l, 0 < l \leq k \\
r^l_{j,0} &= 1, && l < j \leq m, 0 \leq l \leq k \\
R^0_i &= \texttt{shl}(R^0_{i-1}) \texttt{ or } D[t_i], && 0 < i \leq n \\
R^l_i &= (\texttt{shl}(R^l_{i-1}) \texttt{ or } D[t_i]) && \\
&\quad \texttt{and shl}(R^{l-1}_{i-1} \texttt{ and } R^{l-1}_i) && \\
&\quad \texttt{and } (R^{l-1}_{i-1} \texttt{ or } V), && 0 < i \leq n, 0 < l \leq k
\end{aligned}
$$

$$
V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix},
$$

where $v_m = 1$ and $v_j = 0, \forall j, 1 \leq j < m$.

Matrices R^0 and R^1

R^0	$-$	a	d	c
a	1	0	1	1
d	1	1	0	1
b	1	1	1	1
b	1	1	1	1
c	1	1	1	1
a	1	1	1	1

R^1	$-$	a	d	c
a	0	0	0	0
d	1	0	0	0
b	1	1	0	0
b	1	1	1	1
c	1	1	1	1
a	1	1	1	1

D	a	b	c	d	$A \setminus \{a, b, c, d\}$
a	0	1	1	1	1
d	1	1	1	0	1
b	1	0	1	1	1
b	1	0	1	1	1
c	1	1	0	1	1
a	0	1	1	1	1

Matrices R^1 and R^2

R^1	$-$	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a
a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
d	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0
b	1	1	0	0	1	0	1	1	1	0	1	0	0	0	1	1
b	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1
c	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0

R^2	$-$	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a
a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
d	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
b	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
b	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0
c	1	1	1	0	1	1	0	1	1	1	1	1	0	0	0	0
a	1	1	1	1	0	1	1	0	1	1	1	1	1	0	0	0

“Shift-or” revisited

Exact pattern matching

$$
\begin{aligned}
r^0_{j,0} &= 1, & 0 < j \le m \\
R^0_i &= \mathtt{shl}(R^0_{i-1})\ \mathtt{or}\ D[t_i], & 0 < i \le n
\end{aligned}
$$

$$
\begin{aligned}
R^0_1 &= \mathtt{shl}(R^0_0)\ \mathtt{or}\ D[t_1] \\
R^0_2 &= \mathtt{shl}(R^0_1)\ \mathtt{or}\ D[t_2] = \mathtt{shl}(\mathtt{shl}(R^0_0)\ \mathtt{or}\ D[t_1])\ \mathtt{or}\ D[t_2] = \\
&= \mathtt{shl}^2(R^0_0)\ \mathtt{or}\ \mathtt{shl}(D[t_1])\ \mathtt{or}\ D[t_2] \\
R^0_3 &= \mathtt{shl}(R^0_2)\ \mathtt{or}\ D[t_3] = \mathtt{shl}(\mathtt{shl}(R^0_1)\ \mathtt{or}\ D[t_2])\ \mathtt{or}\ D[t_3] = \\
&= \mathtt{shl}(\mathtt{shl}(\mathtt{shl}(R^0_0)\ \mathtt{or}\ D[t_1])\ \mathtt{or}(D[t_2]))\ \mathtt{or}\ D[t_3] = \\
&= \mathtt{shl}^3(R^0_0)\ \mathtt{or}\ \mathtt{shl}^2(D[t_1])\ \mathtt{or}\ \mathtt{shl}(D[t_2])\ \mathtt{or}\ D[t_3]
\end{aligned}
$$

“Shift-or” revisited

$$R_i^0 = \mathtt{shl}^i(R_0^0)\ \mathtt{or}\ \mathtt{shl}^{i-1}(D[t_1])\ \ldots\ \mathtt{or}\ \mathtt{shl}(D[t_{i-1}])\ \mathtt{or}\ D[t_i],$$
$$1 \leq i \leq n$$

Example

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
R^0	–	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	or
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**	
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	
2	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	
3	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	
4	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**0**	**1**	**1**	
5	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	0
6	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**0**	**1**	**1**	0	1
7	0	0	0	0	0	0	0	0	**0**	**1**	**1**	**1**	**1**	**1**	0	0	1
8	0	0	0	0	0	0	0	**0**	**1**	**0**	**1**	**1**	**1**	0	0	0	1
9	0	0	0	0	0	0	**1**	**1**	**1**	**0**	**1**	**1**	0	0	0	0	1
10	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**0**	0	0	0	0	0	1
11	0	0	0	0	**0**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	1
12	0	0	0	**1**	**1**	**0**	**1**	**1**	**0**	0	0	0	0	0	0	0	1
13	0	0	**1**	**1**	**1**	**0**	**0**	**0**	0	0	0	0	0	0	0	0	1
14	0	**0**	**0**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	1
15	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	
16	**1**	**1**	**1**	**0**	**0**	0	0	0	0	0	0	0	0	0	0	0	
17	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	
18	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	
19	**1**	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
20	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

D	a	b	c	d	$A \setminus \{a, b, c, d\}$
a	0	1	1	1	1
d	1	1	1	0	1
b	1	0	1	1	1
b	1	0	1	1	1
c	1	1	0	1	1
a	0	1	1	1	1

R^0	–	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a
a	1	0	1	1	0	1	1	0	0	1	0	1	1	1	1	0
d	1	1	0	1	1	1	1	1	1	1	1	0	1	1	1	1
b	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
b	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Edit distance - table correction

R^0	$-$	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	or
5	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	0
6	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**0**	**1**	**1**	0	1
7	0	0	0	0	0	0	0	0	**0**	**1**	**1**	**1**	**1**	**1**	0	0	1
8	0	0	0	0	0	0	0	**0**	**1**	**0**	**1**	**1**	**1**	0	0	0	1
9	0	0	0	0	0	0	**1**	**1**	**1**	**0**	**1**	**1**	0	0	0	0	1
10	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**0**	0	0	0	0	0	1
11	0	0	0	0	**0**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	1
12	0	0	0	**1**	**1**	**0**	**1**	**1**	**0**	0	0	0	0	0	0	0	1
13	0	0	**1**	**1**	**1**	**0**	**0**	**0**	0	0	0	0	0	0	0	0	1
14	0	**0**	**0**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	1

C^0	$-$	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	or
5	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	0
6	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	1	1
7	0	0	0	0	0	0	0	0	**0**	**1**	**1**	**1**	**1**	**1**	1	1	1
8	0	0	0	0	0	0	0	**0**	**1**	**1**	**1**	**1**	**1**	1	1	1	1
9	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1
10	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1	1
11	0	0	0	0	**0**	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1
12	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1	1
13	0	0	**1**	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1
14	0	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1	1

$$R_i^0 \leftarrow R_i^0 \; or \; 2^{\lceil log_2 R_i^0 \rceil}, \quad m \leq i \leq n$$

$$log_2 x : \mathtt{FYL2X}$$

Hamming distance - replace allowed

$$
\begin{array}{lcll}
r^l_{j,0} & = & 1, & 0 < j \leq m, 0 \leq l \leq k \\
R^0_i & = & \mathtt{shl}(R^0_{i-1}) \text{ or } D[t_i], & 0 < i \leq n \\
R^l_i & = & (\mathtt{shl}(R^l_{i-1}) \text{ or } D[t_i]) \text{ and } \mathtt{shl}(R^{l-1}_{i-1}), & 0 < i \leq n, 0 < l \leq k
\end{array}
$$

$$
\begin{aligned}
R^0_i = \ & \mathtt{shl}^i(R^0_0) \text{ or } \mathtt{shl}^{i-1}(D[t_1]) \ \ldots \\
& \text{ or } \mathtt{shl}(D[t_{i-1}]) \text{ or } D[t_i], 1 \leq i \leq n
\end{aligned}
$$

$$
\begin{aligned}
R^l_i = \ & \mathtt{shl}^i(R^l_0) \text{ or } (\mathtt{shl}^{i-1}(D[t_1]) \text{ and } C^{l-1}_0) \ \ldots \\
& \text{or } (\mathtt{shl}(D[t_{i-1}]) \text{ and } C^{l-1}_{i-2}) \\
& \text{or } (D[t_i] \text{ and } C^{l-1}_{i-1}), \ 1 \leq i \leq n, \ 1 \leq l \leq k
\end{aligned}
$$

Hamming distance - example

C^0	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	or
5	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	0
6	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	1	1
7	0	0	0	0	0	0	0	**0**	**1**	**1**	**1**	**1**	**1**	1	1	1
8	0	0	0	0	0	0	**0**	**1**	**1**	**1**	**1**	**1**	1	1	1	1
9	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1
10	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1	1
11	0	0	0	**0**	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1
12	0	0	**1**	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1	1
13	0	**1**	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1
14	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1	1
R^1	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	or
5	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	0
6	0	0	0	0	0	0	0	0	**0**	**1**	**1**	**0**	**1**	**1**	1	1
7	0	0	0	0	0	0	0	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1
8	0	0	0	0	0	0	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1
9	0	0	0	0	0	**0**	**1**	**1**	**0**	**1**	**1**	1	1	1	1	1
10	0	0	0	0	**0**	**1**	**1**	**1**	**1**	**0**	1	1	1	1	1	1
11	0	0	0	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1
12	0	0	**0**	**1**	**0**	**1**	**1**	**0**	1	1	1	1	1	1	1	1
13	0	**0**	**1**	**1**	**0**	**0**	**0**	1	1	1	1	1	1	1	1	1
14	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1	1

D	a	b	c	d	$A \setminus \{a, b, c, d\}$
a	0	1	1	1	1
d	1	1	1	0	1
b	1	0	1	1	1
b	1	0	1	1	1
c	1	1	0	1	1
a	0	1	1	1	1

Levenshtein distance replace, insert, delete

$$
\begin{aligned}
r^l_{j,0} &= 0, && 0 < j \le l, 0 < l \le k \\
r^l_{j,0} &= 1, && l < j \le m, 0 \le l \le k \\
R^0_i &= \mathtt{shl}(R^0_{i-1}) \ \mathtt{or} \ D[t_i], && 0 < i \le n \\
R^l_i &= (\mathtt{shl}(R^l_{i-1}) \ \mathtt{or} \ D[t_i]) && \\
& \quad \mathtt{and} \ \mathtt{shl}(R^{l-1}_{i-1} \ \mathtt{and} \ R^{l-1}_i) && \\
& \quad \mathtt{and} \ (R^{l-1}_{i-1} \ \mathtt{or} \ V), && 0 < i \le n, 0 < l \le k
\end{aligned}
$$

$$
\begin{aligned}
R^0_i = \ & \mathtt{shl}^i(R^0_0) \ \mathtt{or} \ \mathtt{shl}^{i-1}(D[t_1]) \ \ldots \\
& \mathtt{or} \ \mathtt{shl}(D[t_{i-1}]) \ \mathtt{or} \ D[t_i], 1 \le i \le n
\end{aligned}
$$

$$
\begin{aligned}
R^l_i = \ & \mathtt{shl}^i(R^l_0) \\
& \mathtt{or} \ (\mathtt{shl}^{i-1}(D[t_1]) \ \mathtt{and} \ C^{l-1}_0 \ \mathtt{and} \ \mathtt{shl}(C^{l-1}_1) \ \mathtt{and} \ (\mathtt{shr}(C^{l-1}_0) \ \mathtt{or} \ V)) \\
& \ldots \\
& \mathtt{or} \ (\mathtt{shl}(D[t_{i-1}] \ \mathtt{and} \ C^{l-1}_{i-2} \ \mathtt{and} \ \mathtt{shl}(C^{l-1}_{i-1}) \ \mathtt{and} \ (\mathtt{shr}(C^{l-1}_{i-2}) \ \mathtt{or} \ V)) \\
& \mathtt{or} \ (D[t_i] \ \mathtt{and} \ C^{l-1}_{i-1} \ \mathtt{and} \ \mathtt{shl}(C^{l-1}_i) \ \mathtt{and} \ (\mathtt{shr}(C^{l-1}_{i-1}) \ \mathtt{or} \ V)), \\
& 1 \le i \le n, \ 1 \le l \le k
\end{aligned}
$$

Example

C^0	$-$	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	or
3	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	
4	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
7	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
8	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
9	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
10	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
11	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
12	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

R^1	$-$	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	or
3	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
7	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
8	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
9	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1	1
10	0	0	0	0	0	0	1	1	1	1	0	1	1	1	1	1	1
11	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	1	1	0	1	1	1	1	1	1	1	1
13	0	0	0	0	1	0	0	0	1	1	1	1	1	1	1	1	1
14	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
15	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1

D	a	b	c	d	$A \setminus \{a,b,c,d\}$
a	0	1	1	1	1
d	1	1	1	0	1
b	1	0	1	1	1
b	1	0	1	1	1
c	1	1	0	1	1
a	0	1	1	1	1

$\nearrow$I $\rightarrow$R $\downarrow$D

Example

R^1	$-$	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	or
3	0	0	0	0	0	0	0	0	0	0	0	0	**0**	**1**	**1**	**1**	
4	0	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	
5	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	0
6	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	1	0
7	0	0	0	0	0	0	0	0	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1
8	0	0	0	0	0	0	0	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1
9	0	0	0	0	0	0	**0**	**0**	**1**	**0**	**1**	**1**	1	1	1	1	1
10	0	0	0	0	0	**0**	**1**	**1**	**1**	**1**	**0**	1	1	1	1	1	1
11	0	0	0	0	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1
12	0	0	0	**0**	**0**	**0**	**1**	**1**	**0**	1	1	1	1	1	1	1	1
13	0	0	**0**	**0**	**1**	**0**	**0**	**0**	1	1	1	1	1	1	1	1	1
14	0	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1	1
15	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1	1	1
16	**1**	**1**	**1**	**0**	**0**	1	1	1	1	1	1	1	1	1	1	1	1

C^1	$-$	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	or
3	0	0	0	0	0	0	0	0	0	0	0	0	**0**	**1**	**1**	**1**	
4	0	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	
5	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	0
6	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	1	0
7	0	0	0	0	0	0	0	0	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1
8	0	0	0	0	0	0	0	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1
9	0	0	0	0	0	0	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1	1	1
10	0	0	0	0	0	**0**	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1	1
11	0	0	0	0	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1
12	0	0	0	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1	1	1	1	1
13	0	0	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1
14	0	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1	1
15	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1	1	1
16	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1	1	1	1

D	a	b	c	d	$A \setminus \{a,b,c,d\}$
a	0	1	1	1	1
d	1	1	1	0	1
b	1	0	1	1	1
b	1	0	1	1	1
c	1	1	0	1	1
a	0	1	1	1	1

$\nearrow$I $\rightarrow$R $\downarrow$D

Example

C^1	–	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	or
3	0	0	0	0	0	0	0	0	0	0	0	0	**0**	**1**	**1**	**1**	
4	0	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	
5	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	0
6	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	1	0
7	0	0	0	0	0	0	0	0	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1
8	0	0	0	0	0	0	0	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1
9	0	0	0	0	0	0	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1	1	1
10	0	0	0	0	0	**0**	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1	1
11	0	0	0	0	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1
12	0	0	0	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1	1	1	1	1
13	0	0	**0**	**0**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1
14	0	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1	1
15	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1	1	1
16	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1	1	1	1	1	1

R^2	–	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	or
3	0	0	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	
4	0	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	
5	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	0
6	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	1	0
7	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	1	1	0
8	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**1**	**1**	1	1	1	1
9	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**1**	**1**	1	1	1	1	1
10	0	0	0	0	0	**0**	**0**	**0**	**1**	**1**	**0**	1	1	1	1	1	1
11	0	0	0	0	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1	1	1	1
12	0	0	0	**0**	**0**	**0**	**0**	**1**	**0**	1	1	1	1	1	1	1	1
13	0	0	**0**	**0**	**0**	**0**	**0**	**0**	1	1	1	1	1	1	1	1	0
14	0	**0**	**0**	**0**	**0**	**1**	**1**	1	1	1	1	1	1	1	1	1	1
15	**0**	**0**	**0**	**0**	**1**	**1**	1	1	1	1	1	1	1	1	1	1	1
16	**0**	**0**	**0**	**0**	**0**	1	1	1	1	1	1	1	1	1	1	1	0

D	a	b	c	d	$A \setminus \{a,b,c,d\}$
a	0	1	1	1	1
d	1	1	1	0	1
b	1	0	1	1	1
b	1	0	1	1	1
c	1	1	0	1	1
a	0	1	1	1	1

$\nearrow$I $\rightarrow$R $\downarrow$D

Example

R^2	$-$	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	or
3	0	0	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	
4	0	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	
5	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	0
6	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	1	0
7	0	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	1	1	0
8	0	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**1**	**1**	1	1	1	1
9	0	0	0	0	0	0	**0**	**0**	**0**	**0**	**1**	**1**	1	1	1	1	1
10	0	0	0	0	0	**0**	**0**	**0**	**1**	**1**	**0**	1	1	1	1	1	1
11	0	0	0	0	**0**	**0**	**0**	**1**	**1**	**1**	1	1	1	1	1	1	1
12	0	0	0	**0**	**0**	**0**	**0**	**1**	**0**	1	1	1	1	1	1	1	1
13	0	0	**0**	**0**	**0**	**0**	**0**	**0**	1	1	1	1	1	1	1	1	0
14	0	**0**	**0**	**0**	**0**	**1**	**1**	1	1	1	1	1	1	1	1	1	1
15	**0**	**0**	**0**	**0**	**1**	**1**	1	1	1	1	1	1	1	1	1	1	1
16	**0**	**0**	**0**	**0**	**0**	1	1	1	1	1	1	1	1	1	1	1	0

R^1	$-$	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a
a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
d	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0
b	1	1	0	0	1	0	1	1	1	0	1	0	0	0	1	1
b	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1
c	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0

R^2	$-$	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a
a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
d	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
b	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
b	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0
c	1	1	1	0	1	1	0	1	1	1	1	1	0	0	0	0
a	1	1	1	1	0	1	1	0	1	1	1	1	1	0	0	0

D	a	b	c	d	$A \setminus \{a,b,c,d\}$
a	0	1	1	1	1
d	1	1	1	0	1
b	1	0	1	1	1
b	1	0	1	1	1
c	1	1	0	1	1
a	0	1	1	1	1

$\nearrow$I $\rightarrow$R $\downarrow$D

Complexities

- $\mathcal{O}(1)$ time for the exact pattern matching
- $\mathcal{O}(k)$ for the approximate pattern matching
- $\mathcal{O}(n)$ processors
- *CREW PRAM*

Future work

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
R^0	–	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	or
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	
2	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	
3	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	
4	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	1	1	1	0	1	1	0	1
7	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	1
8	0	0	0	0	0	0	0	0	1	0	1	1	1	0	0	0	1
9	0	0	0	0	0	0	1	1	1	0	1	1	0	0	0	0	1
10	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	1
11	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	1
12	0	0	0	1	1	0	1	1	0	0	0	0	0	0	0	0	1
13	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1
14	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	1
15	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	
16	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	
17	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	
18	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	
19	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
20	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

R^0	–	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	nul	nul
0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1	1	0	0	0	0	1	1	1	0	0	0	0	0	0
3	0	0	1	1	1	0	0	0	0	1	1	1	0	0	1	1	0	0
4	0	0	0	1	1	1	0	0	1	0	1	1	0	1	1	1	0	0
5	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0
6	1	1	1	0	0	0	1	1	1	1	0	0	1	0	1	1	0	0
7	1	1	1	1	0	0	1	1	1	1	0	0	0	0	0	0	0	0
8	1	1	1	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0
9	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
10	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0

R^0	–	a	d	c	a	b	c	a	a	b	a	d	b	b	c	a	nul	nul
0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	0	0	0
3	0	0	1	1	1	1	0	0	0	1	1	1	0	0	1	1	1	1
4	0	0	0	1	1	1	0	0	1	1	1	1	0	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Complexities

- $\mathcal{O}(log\ m)$ parallel time
- $\mathcal{O}(\frac{n}{log\ m})$ processors
- *EREW PRAM*
- Based on bit-parallel *NFA* simulation
- Linear memory