PSC’06
On Implementation and Performance of Table-driven DFA-based String Processors

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Agenda

1. Introduction
2. Implementation Strategies
3. Characterization of TD Recognizers
4. The various Table-driven Algorithms
5. Experimental Results
6. Conclusion and Future Work
Introduction

DFA-based String Recognition: The problem

Given an automaton $M(\mathcal{Q}, \mathcal{V}, \delta, \mathcal{F}, s_0)$, where:
- $\mathcal{Q}$ is the set of states;
- $\mathcal{V}$ the alphabet;
- $\delta$ the transition function;
- $\mathcal{F} \subseteq \mathcal{Q}$ the set of final states;
- $s_0$ the start state;

Such that $\mathcal{L}(M)$ is the language modelled by $M$ with $\mathcal{L}(M) \subseteq \mathcal{V}^*$;
And $s$, an input string ($s \in \mathcal{V}^*$)
Check whether $s \in \mathcal{L}(M)$
Introduction

Implementation:

Hardcoding:
  Transition table embedded in the algorithm
  Useful for automata of relatively small size (in the order of hundreds)
Introduction

Implementation:

Table-driven:
- Rows represent states and columns alphabet symbols
- Performance determined by the pattern of accesses in the table
- Efficient if rows are accessed contiguously
- High probability of cache misses if random pattern of accesses
Introduction

- Example of TD DFA-based Recognizer:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
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</thead>
<tbody>
<tr>
<td>0</td>
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</tbody>
</table>

The string $s_1 = aaaaa$ is processed faster than the string $s_2 = bcde$

For $s_1$ rows are access contiguously, hence high probability of cache hits
High probability of cache misses for $s_2$

- Problem: explore various strategies aimed at lowering probability of cache misses during acceptance testing
### Implementation Strategies

- **Dynamic State Allocation (DSA)**

  A dynamically allocated state is created in memory for acceptance testing. Copy into a block of memory first encountered states. Subsequent reference to such state is made via the new piece of memory.

  - **Example: accepting the string** $bced$

<table>
<thead>
<tr>
<th></th>
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</tbody>
</table>

  - **Will result in:**

<table>
<thead>
<tr>
<th></th>
<th>a</th>
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<td>3</td>
<td>-I</td>
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<td>0</td>
</tr>
</tbody>
</table>
Implementation Strategies

- **Dynamic State Allocation (DSA)**
  - More details are provided in PSC’05
  - Relies on the use of a natural number $D$
    * Determines the extent of the dynamic allocation of memory blocks

\[
\begin{align*}
D = 0 & \equiv \text{no DSA strategy} \\
D = |Q| & \equiv \text{the strategy is \textit{unbounded}} \\
D < |Q| & \equiv \text{the strategy is \textit{bounded}}
\end{align*}
\]

May require replacement policy when threshold is reached
- Direct mapping policy
- LRU policy
- Associative mapping
- No policy
Implementation Strategies

- State pre-Ordering (SpO)

- Very low percentage of visited states compared to the overall DFA size
- Frequently visited states scattered throughout the transition table

  - Reorganize $\delta$ such that frequently accessed states are grouped together
  - Reduces the probability of cache misses
  - Reorders the DFA’s original placement of states in the transition table

Relies on user’s input:
- A priori reasoning
- Empirical analysis of the history of states visits in prior runs
Implementation Strategies

- **State pre-Ordering (SpO)**

A boolean argument $P$ is passed to the function such that:

$P = F \equiv \text{SpO strategy not used}$

$P = T \equiv \text{preprocessing operation required for reordering}$

- Access to state $i$ is done via an array $p[0..n]$
- The $i^{th}$ entry is the new row of $i$ in the transition table

- **Example:** $P = T, p[0..3] = \{0, 3, 2, 1\}$

- Before acceptance testing, the table is reordered into:
Implementation Strategies

- Allocated Virtual Caching (AVC)

*Treats a portion of the memory that holds the transition table as a cache*

*The first $V$ rows of the table acts as a virtual cache*
- Virtually acts as the hardware cache memory (Replacement policies)

*Hope to enhance cache’s spatial and temporal locality of reference*
- Individual states are transferred into the cache as they are visited
- Virtual cache limited in size
- Reference to a state out of the cache when it is full requires replacement
- The cache initially is regarded as empty
- Use of a pointer to manage cache utilization
Implementation Strategies

- Allocated Virtual Caching (AVC)

Example:

\[ V = 1 \text{ (cache size)}; \]
\[ c = \{0, 1\} \text{ (states initially in cache)} \]
\[ l = 0 \text{ (pointer)}; \]
\[ i = \{F', F', F', F'\} \text{ (cache indicator)} \]
\[ q = 0 \text{ (start state)}; \]
\[ m = \{0, 1, 2, 3\} \text{ (current states’ position)} \]
\[ s = bcdc \text{ (string to be tested)} \]

The table initially:

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Implementation Strategies

- Allocated Virtual Caching (AVC)

We encounter the symbol $b$

\[
\begin{align*}
l &= 0 \\ q &= 0 \\ c_l &= q = 0
\end{align*}
\]

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Acceptance testing:

\[
\begin{align*}
l &= l + 1 = 1 \\ c &= \{0, 1\} \\ i &= \{T, F, F, F\} \\ m &= \{0, 1, 2, 3\} \\ q &= \delta(q, b) = 3
\end{align*}
\]
Implementation Strategies

- Allocated Virtual Caching (AVC)

We encounter the symbol $c$

\[
\begin{align*}
l &= 1 \leq V \\
q &= 3 \\
c_l &= 1 \neq q = 3
\end{align*}
\]

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</tbody>
</table>

Acceptance testing:

\[
l = l + 1 = 2 > V \text{ (the cache is full)}
\]

\[
c = \{0, 3\}
\]

\[
i = \{T, F, F, T\}
\]

\[
m = \{0, 3, 2, 1\}
\]

\[
q = \delta(m_q, c) = 2
\]
Implementation Strategies

- Allocated Virtual Caching (AVC)

We encounter the symbol $d$

$l = 2 > V$

$q = 2$

$i_q = F$

The cache is full:
- Replacement policy to remove a state from the cache;
- Do acceptance testing without any further replacement

The other iterations are easy to follow
Characterization of TD Recognizers

- **Definition:**

A string recognizer of a DFA is an algorithm that relies on the DFA’s transition function to determine whether a string is part of the language modelled by the DFA or not.
Characterization of TD Recognizers

- **Definition:**

  Given an input string $s$, an automaton $M(Q, V, \delta, F, s_0)$, the recognizer scans each symbol of $s$ and returns a boolean $\mathbb{B} = \{T, F\}$

  *The definition is somewhat general*
  
  No restriction on how the recognizer would be implemented
  ○ Need to introduce the strategies previously described

  We consider a recognizer as a function $\rho$ that accepts as arguments $s, \delta$ and the respective strategy variables, and returns a boolean:
  ○ $\rho$ is the *denotational semantics* of the recognizer
Characterization of TD Recognizers

- **Formalism:**

  Specification of $\rho$ in functional terms
  Assume $T = Q \times V$ (transition relation)

  $$\rho : T \times N \times B \times N \times V^* \to B$$

  $$\rho(\delta, D, P, V, s) = \begin{cases} 
  T & \text{if } s \in \mathcal{L}(M) \\
  F & \text{if } s \notin \mathcal{L}(M) 
  \end{cases}$$

  where:
  
  $0 \leq D \leq |Q|$ (DSA strategy);
  $P \in B$ (SpO strategy);
  $0 \leq V < |Q|$ (AVC strategy).
The various TD Algorithms

- Various instances:

DSA Strategy: The variable $D \in \{0, d, |Q|\}; d < |Q|$
SpO Strategy: The variable $P \in \{T, F\}$
AVC Strategy: The variable $V \in \{0, v\}; v < |Q|$

Resulting in $3 \times 2 \times 2 = 12$ different algorithms

<table>
<thead>
<tr>
<th>Combination</th>
<th>Active strategy</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, F, 0)</td>
<td>None (core TD)</td>
<td>$t$</td>
</tr>
<tr>
<td>(d, F, 0)</td>
<td>bounded DSA</td>
<td>$t_{b1}$</td>
</tr>
<tr>
<td>(n, F, 0)</td>
<td>unbounded DSA</td>
<td>$t_{u1}$</td>
</tr>
<tr>
<td>(0, T, 0)</td>
<td>SpO</td>
<td>$t_2$</td>
</tr>
<tr>
<td>(0, F, v)</td>
<td>AVC</td>
<td>$t_3$</td>
</tr>
<tr>
<td>(0, T, v)</td>
<td>SpO and AVC</td>
<td>$t_{23}$</td>
</tr>
<tr>
<td>(d, T, 0)</td>
<td>bounded DSA and SpO</td>
<td>$t_{b12}$</td>
</tr>
<tr>
<td>(d, T, v)</td>
<td>bounded DSA, SpO and AVC</td>
<td>$t_{b123}$</td>
</tr>
<tr>
<td>(d, F, v)</td>
<td>bounded DSA and AVC</td>
<td>$t_{b13}$</td>
</tr>
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<tr>
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<td>unbounded DSA and AVC</td>
<td>$t_{u13}$</td>
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</table>
The various TD Algorithms

- The core TD algorithm

Takes as input the transition function $\delta$, the string $s$ and returns a boolean

**Algorithm 0.1** (The core table-driven recognizer)

```
proc $t(\delta, s)$
    ; $q, j := 0, 0$
    do $(j < s\text{.len}) \land (q \geq 0) \rightarrow$
        $q, j := \delta(q, s_j), j + 1$
    od
    if $q < 0 \rightarrow \{\text{return F}\} \parallel q \geq 0 \rightarrow \{\text{return T}\}$ fi
```
The various TD Algorithms

- The bounded TD-DSA Algorithm ($t_{b1}$)

The strategy argument $D < |Q|$

○ Unbounded counterpart was discussed in PSC’05

May involve state replacement

when reference is made to a state that has not yet been visited

provided the allocated dynamic space is full
The various TD Algorithms

- The bounded TD-DSA Algorithm \((t_{b1})\)

**Algorithm 0.2** (The bounded TD-DSA recognizer)

\[
\text{proc } t_{b1}(\delta, D, A, Z, s) \\
; m[0..n], B, q, j, p := -1, A, 0, 0 \\
; \text{do } (j < s\text{.len} \wedge q \geq 0) \rightarrow \\
\quad \text{if } (m_q = -1) \rightarrow \{ \text{state not dynamically allocated} \} \\
\quad \text{if } (p < D) \rightarrow \\
\quad \quad ; m_q, d_p := p, \text{malloc}(B, Z) \\
\quad \quad ; d_{p,[0..a]}, p, B := \delta_q,[0..a], p + 1, B + Z \\
\quad \quad ; q := d_{p,s_j} \\
\quad \quad \| (p \geq D) \rightarrow \\
\quad \quad \quad r := \text{MOD}(q, D) \{ \text{remainder in the division of } q \text{ by } D \} \\
\quad \quad \quad ; m_q := r \\
\quad \quad \quad ; i := \text{search}(m, r) \\
\quad \quad \quad ; m_i := -1 \\
\quad \quad \quad ; d_{r,[0..a]}, q := \delta_q,[0..a], d_{r,s_j} \\
\quad \quad \text{fi} \\
\quad \quad \| m_q \neq -1 \rightarrow \text{skip} \{ \text{state dynamically allocated} \} \\
\quad \text{fi} \\
\quad ; q, j := d_{m_q,s_j}, j + 1 \\
\text{od} \\
\text{if } q < 0 \rightarrow \{ \text{return F} \} \| q \geq 0 \rightarrow \{ \text{return T} \} \text{ fi}
\]
The various TD Algorithms

• The TD-SpO algorithm

Additional input: auxiliary array $p_{[0..n]}$

$p_i$ is the new row of $\delta$ where $i^{th}$ row of $\delta$ should be moved

$\text{reorder}(\delta, p)$ recorders the DFA’s states according to $p$’s entries
Access to a state $q$ is made indirectly via $p$:
i.e. $q = \delta(p_p, s_j)$
The various TD Algorithms

- The TD-SpO algorithm

**Algorithm 0.3 (The TD-SpO recognizer)**

```plaintext
proc t_2(\delta, p, s)
  ; reorder(\delta, p)
  ; q, j := 0, 0
  do (j < s.len) \land (q \geq 0) \rightarrow
    q, j := \delta(p_q, s_j), j + 1
  od
  if q < 0 \rightarrow \{return F\} \parallel q \geq 0 \rightarrow \{return T\} fi
```
The various TD Algorithms

- The TD-AVC algorithm

\[ V: \text{the size of the virtual cache} \]
\[ m_{[0..n]}: \text{holds current state position in } \delta \]
\[ c_{[0..V]}: \text{holds states currently in the virtual cache} \]
\[ i_{[0..n]}: \text{holds information indicating whether a state is in cache or not} \]
\[ l: \text{cache line controller. } l = 0 \Rightarrow \text{cache is empty; } l \geq V \Rightarrow \text{cache is full} \]

**Acceptance testing:**
- if \( l \leq V \)
  - Acceptance testing if the current state matches the current cache line
  - Otherwise, invoke \( swd(\delta[m_q], \delta[m_c]) \) before acceptance testing
- if \( l > V \)
  - Use replacement policy before acceptance testing
The various TD Algorithms

- The TD-AVC algorithm

Algorithm 0.4 (Table-driven based on allocated virtual caching)

```plaintext
proc t3(δ, V, s)
; q, j, p, l := 0, 0, 0, 0
; m[0..n], e[0..V], i[0..n] := [0..n], [0..V], -1
do (j < s.len) ∧ (q ≥ 0) →
  if (i_q ≠ -1) → skip
    → (i_q = -1) ∧ (l < V) →
      if q = c_l → skip
        → q ≠ c_l →
          p := c_l
          ; swd(δ[m_q], δ[m_p])
          ; i_p, c_l := -1, q
      fi
    ; i_q, l := 0, l + 1
  (i_q = -1) ∧ (l ≥ V) →
  p := MOD(m_q, V)
  swd(δ[m_q], δ[m_c_p])
  i_q, i_c_p, c_p := 0, -1, q
fi
; q, j := δ(m_q, s_j), j + 1
od
if q < 0 → {return F} || q ≥ 0 → {return T} fi
```
The various TD Algorithms

- The TD-SpO-AVC algorithm

Relies on both SpO and AVC

A naïve approach:
- Use $\text{reorder}(\delta, p)$ to reorder the automaton’s states (SpO)
- Perform acceptance testing using AVC approach
The various TD Algorithms

- The TD-SpO-AVC algorithm

**Algorithm 0.5 (The TD-SpO-AVC algorithm)**

```
proc t_{23}(\delta, p, V, s)
; reorder(\delta, p)
; q, j, p, l := 0, 0, 0, 0
; m_{[0..n]}, c_{[0..V]}, i_{[0..n]} := [0..n], [0..V], -1
do \((q < s.len) \land (q \geq 0)\) →
   tdavc(\delta, p, m, c, i, l, V, j, q, s)
od
if q < 0 → \{return F\} \parallel q \geq 0 → \{return T\} fi
```
The various TD Algorithms

- The bounded and unbounded TD-DSA-AVC algorithm

Combination of the AVC with either the bounded/unbounded DSA

A simple policy:
- The first $k$ states of the DFA are cacheable with $0 < V < k < n$
- The remaining $n - k$ states are processed using bounded DSA
- Test whether the state is cacheable or not.
The various TD Algorithms

- The bounded and unbounded TD-DSA-AVC algorithm

**Algorithm 0.6** (The unbounded TD-DSA-AVC algorithm)

```
proc \( t_{ul3} (\delta, k, A, Z, s) \)
; \( q, j, p, l, B := 0, 0, 0, 0, A \)
; \( m[0..n], c[0..V], i[0..n] := [0..n], [0..V], -1 \)
; do \((j < s.len \land q \geq 0) \rightarrow \)
\quad if \( q < k \rightarrow tdauc(\delta, m, c, i, l, V, j, q, s) \)
\quad \| \quad q \geq k \rightarrow utddsa(\delta, A, Z, B, q, j, s) \)
\quad fi
\od
\if \( q < 0 \rightarrow \{ \text{return } F \} \) \| \( q \geq 0 \rightarrow \{ \text{return } T \} \) fi
```
The various TD Algorithms

- The bounded and unbounded TD-SpO-DSA-AVC algorithm
  - It is a combination of:
    - SpO strategy;
    - Bounden or Unbounded DSA strategy and
    - AVC strategy.
  - A simple policy for the unbounded TD-SpO-DSA-AVC implementation:
    - ∗ reorder the states in $\delta$ based on entries of an auxiliary array $p[0..n]$
    - ∗ Apply the unbounded TD-DSA-AVC procedure discussed in the previous slide
    - ∗ Access to states information is done indirectly via $p$
The various TD Algorithms

- The bounded and unbounded TD-SpO-DSA-AVC algorithm

**Algorithm 0.7** (The unbounded TD-SpO-DSA-AVC algorithm)

```plaintext
proc \( t_{u123}(\delta, p, s, c, k, d, A, Z) \)
; reorder(\( \delta, p \))
{ Initializations }
;do \((q < s.len \land q \geq 0) \rightarrow \)
  if \( q < k \rightarrow tdavc(\delta, p, m, c, i, l, V, j, q, s) \)
  \| \( q \geq k \rightarrow utddsa(\delta, p, A, Z, B, q, j, s) \)
fi
od
if \( q < 0 \rightarrow \{ \text{return } F \} \) \| \( q \geq 0 \rightarrow \{ \text{return } T \} \) fi
```
Experimental Results

Bounded TD-DSA vs. TD

Unbounded TD-DSA vs. TD
Experimental Results

TD-SpO vs. TD

TD-AVC vs. TD
Experimental Results

Unbounded TD-DSA-SpO vs. TD  bounded TD-DSA-SpO vs. TD
Experimental Results

Bounded TD-DSA-AVC vs. TD

unbounded TD-DSA-AVC vs. TD
Experimental Results

Unbounded TD-DSA-SpO-AVC vs. TD  Bounded TD-DSA-SpO-AVC vs. TD
Experimental Results

TD-AVC vs. TD
Conclusion and Future Work

- We have investigated various ways of improving the performance of DFA-based string processors
  - A 6-arguments function provided the denotational semantics of a string recognizer
  - Instantiations of the arguments resulted in 12 different table-driven DFA-based string recognizers
  - Some algorithms outperformed the core TD algorithm for strings mae of long sequences
  - Some algorithm were not of interest due to the replacement policy used during acceptance testing

As a matter of future work:
  - Need to explore in dept the replacement policy in used
  - need to test the algorithm on real life data