Weak Factor Automata: Comparing (Failure) Oracles and Storacles

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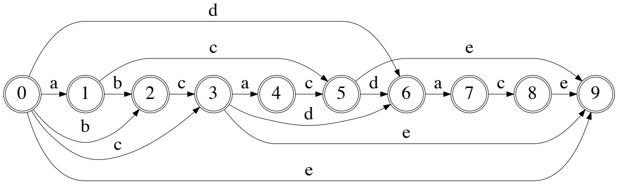
(Weak) factor automata

- Factor automaton (DAWG)
  - Accepts factors of keyword
  - Used for efficient backward pattern matching
  - Used as index
- Weak factor automata
  - (Small) over-approximation
    ... to save space
  - OK for pattern matching
  - May be OK for indexing

Contributions

- New weak factor automata constructions
  - Based on factor oracle and factor storacle
  - Using failure transitions
  - Failure factor oracle
  - Failure factor storacle
- Empirical size comparison
  - On generated strings of lengths $4 - 9$
  - On English word list (lengths $4 - 28$)

Factor oracle



- Small over-approximation—e.g. *bce*, *cace*
- $m+1$ states, m to $2m-1$ transitions
- Acylic
- Homogeneous
- $O(m)$ construction (*Allauzen, Crochemore & Raffinot 1999*)
- $O(m^2)$ construction (*Cleophas, Zwaan & Watson 2003/2005*)
  - Conceptually simpler, some properties obvious

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Factor oracle construction algorithm

for i from 0 to m **do**
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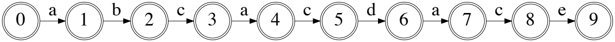

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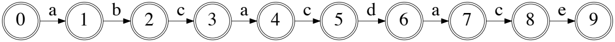

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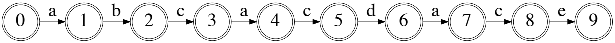

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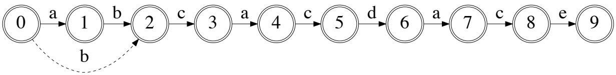

Suffix bcacdace

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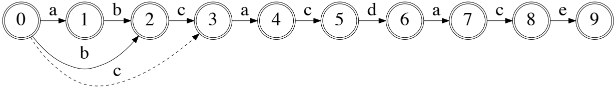

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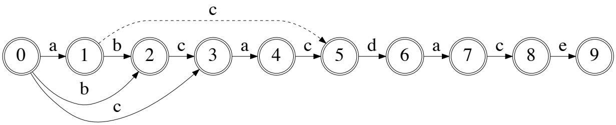
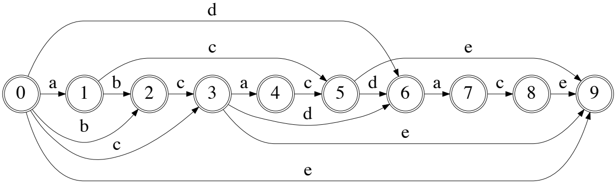


Suffix acdace

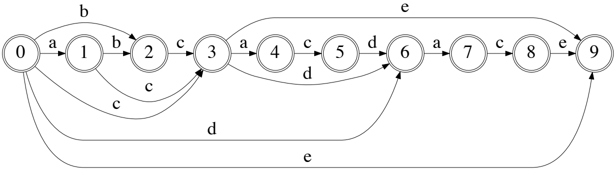
Factor oracle

... etc. leads to



Factor storacle

- Modification to $O(m^2)$ FO construction
- shortest forward **t**ransition **oracle**
  - ... keeping it homogeneous
  - Accidental...
  - Example smaller than FO



- Not smaller than FO in general (*Cleophas & Watson 2012*)

  ... usually *slightly larger*
- Conjecture m to $3m$ transitions

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Factor storacle construction algorithm

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for i from 0 to m do
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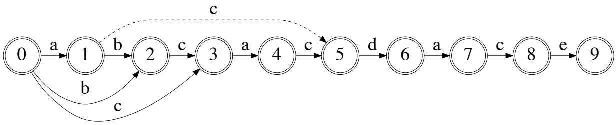
Factor storacle construction example

Recall factor oracle case:



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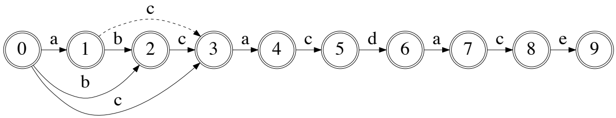

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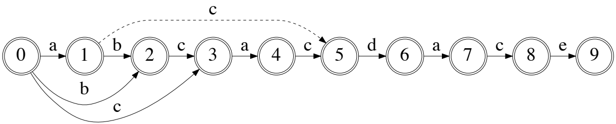

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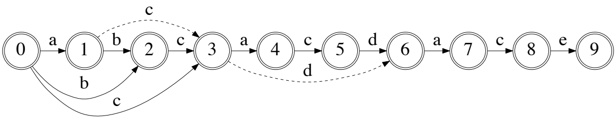

Failure transitions

- Allow failure transitions in addition to symbol ones
- Save space, but more transition use...
- Not new
  - Aho-Corasick
  - Generalized by *Crochemore & Hancart 1997*
- First general *DFA* $\rightarrow$ *FDFA* algorithm by *Kourie et al. 2012*
  - Intermediate lattice construction
    ... keeping state set fixed
- *Björklund et al. 2013*
  - Complexity...
    ... even if keeping state set fixed
  - Algorithm to reach $\frac{2}{3}$ of optimal savings
- Both *ex post facto*...

Our idea

- Introduce failure transitions *during construction*
- We call the resulting automata *Failure Factor (St)Oracles*
- Complexity as for non-failure cases
- Idea: instead of constructing $j \xrightarrow{a} k+1$ ...

  ... construct $j \xrightarrow{fail} k$, from which $k \xrightarrow{a} k+1$ exists
- Potential problem...

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- Potential problem...
- Using sequence of existing (failure, symbol) transitions may end up in $j > k$

Potential problem...

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  ... potential for *backward* failure transition

  ... hence for *cycle*

  ... hence for *failure cycle*

  ... hence for *divergent failure cycle*

  ... leading to live-lock in construction or use of automaton
- Solution: do not construct backward *failure* transition

  ... instead create non-forward *symbol* transition

  ... still potential for cycle (but manageable)

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  if $k \neq m$ **then**
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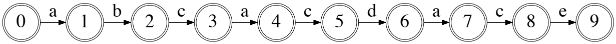
Failure factor storacle construction algorithm similar

Failure factor oracle construction example



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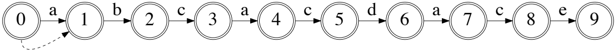

Suffix bcacdace

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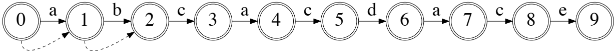

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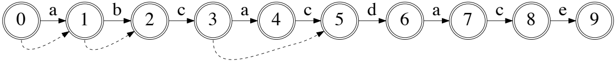

Suffix acdace

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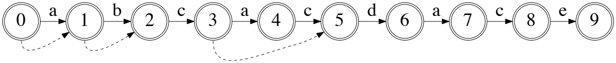
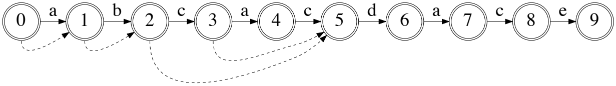


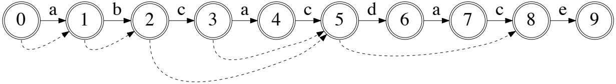
Suffix acdace
Suffix cdace

Failure factor oracle construction example



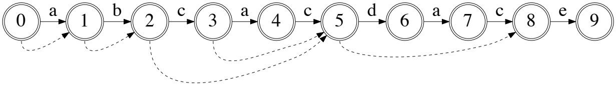
Suffix dace

Failure factor oracle construction example



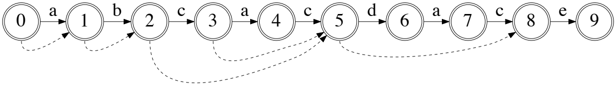
Suffix ace

Failure factor oracle construction example



Suffix ace
Suffix ce

Failure factor oracle construction example



Suffix ace

Suffix ce

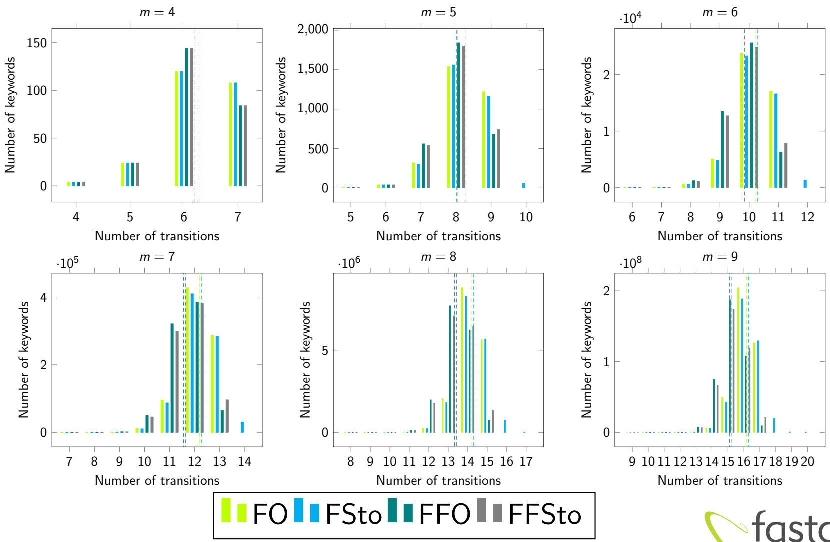
Suffix e

Empirical results

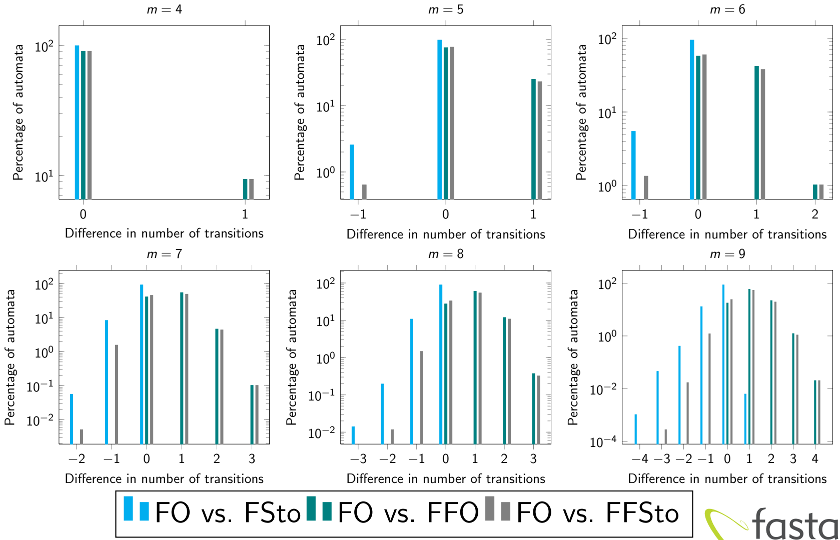
Two data sets

- Generated strings: all strings of length $m \in [4, 9]$ over alphabet of size m
- English words

Generated strings—number of transitions

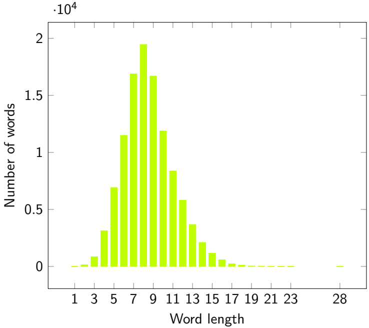


Generated strings—difference in #transitions of FO vs. ...



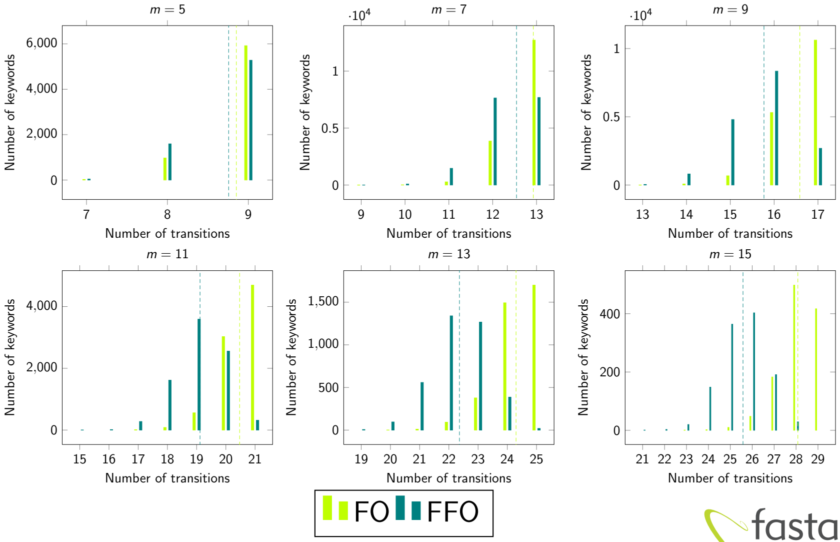
Empirical results on English words

- English word list from `http://www.sil.org/linguistics/wordlists/english`.

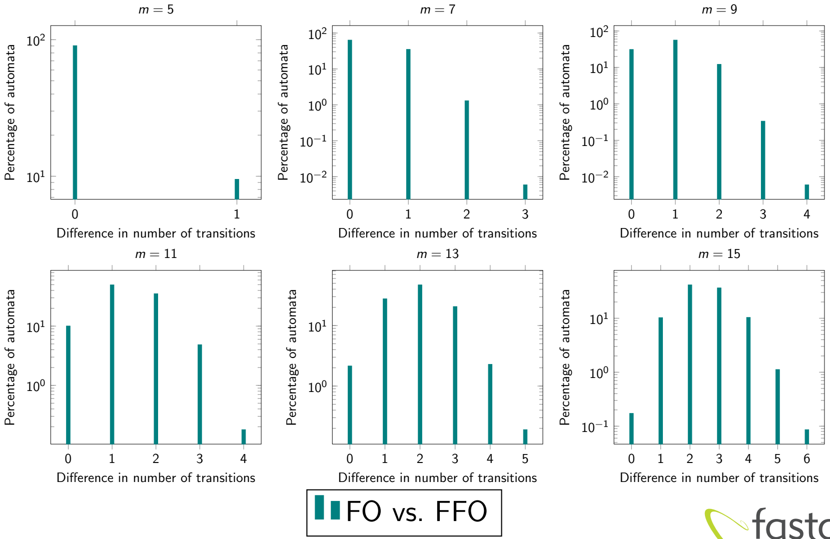


... disregarding words p with $|p| < 4$

English words—number of transitions



English words—difference in #transitions of FO vs. FFO



Concluding remarks

- Failure versions save ca. 2-10% on #transitions

  ... possibly more on space
- Open questions
  - Upper bounds on number of transitions
  - Languages
  - Comparison to general super automata
  - Comparison to general FDFA construction algorithm
- Performance when using automata

  ... recent work: FFO for DNA strings of lengths $4 - 2048$
  - Savings of $8 - 10\%$ for lengths $16 - 2048$
  - Also rudimentary processing; runtime increases $34 - 88\%$

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References

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