Pattern Matching on Weighted Strings

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Prague Stringology Conference 2019

Plan of Presentation

1. Weighted Strings
2. Weighted Pattern Matching and Profile Matching
3. General Weighted Pattern Matching
4. Weighted Indexing
5. On-line and Streaming Weighted Pattern Matching
6. Weighted LCS and SCS

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Strings, Partial Words, Indeterminate Strings

Strings (solid strings):

`a` `c` `a` `b` `b` `b`

Strings, Partial Words, Indeterminate Strings

Strings (solid strings):

$\mathtt{a} \quad \mathtt{c} \quad \mathtt{a} \quad \mathtt{b} \quad \mathtt{b} \quad \mathtt{b}$

Partial words (strings with don't care symbols):

$\mathtt{a} \quad \diamondsuit \quad \mathtt{a} \quad \mathtt{b} \quad \diamondsuit \quad \mathtt{b}$

Strings, Partial Words, Indeterminate Strings

Strings (solid strings):

a	c	a	b	b	b

Partial words (strings with don't care symbols):

a	$\diamondsuit$	a	b	$\diamondsuit$	b
a	c	a	b	b	b
a	a	a	b	a	b
a	b	a	b	c	b

$\vdots$

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Indeterminate strings:

a	b c	a	b	a b	b

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Indeterminate strings:

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a	c	a	b	b	b
a	c	a	b	a	b
a	b	a	b	b	b
a	b	a	b	a	b

Weighted Strings (PPMs) and Profiles (PWMs)

Weighted Strings (Position Probability Matrices):

a	b 0.2 c 0.8	a	b	a 0.6 b 0.4	b

Weighted Strings (PPMs) and Profiles (PWMs)

Weighted Strings (Position Probability Matrices):

a	b 0.2 c 0.8	a	b	a 0.6 b 0.4	b	probability
a	c	a	b	b	b	0.32
a	c	a	b	a	b	0.48
a	b	a	b	b	b	0.08
a	b	a	b	a	b	0.12

Weighted Strings (PPMs) and Profiles (PWMs)

Weighted Strings (Position Probability Matrices):

a 1 b 0 c 0	a 0 b 0.2 c 0.8	a 1 b 0 c 0	a 0 b 1 c 0	a 0.6 b 0.4 c 0	a 0 b 1 c 0	probability
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Profiles (Position Weight Matrices):

a 7	a 3	a 0	a 0	a 6	a 1
b 0	b 2	b 1	b 5	b 4	b 9
c 1	c 8	c 0	c 0	c 3	c 0

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Profiles (Position Weight Matrices):

a 7	a 3	a 0	a 0	a 6	a 1	
b 0	b 2	b 1	b 5	b 4	b 9	score
c 1	c 8	c 0	c 0	c 3	c 0	
a	c	a	b	b	b	33
a	c	a	b	a	b	35
a	b	a	b	b	b	27
a	b	a	b	a	b	29

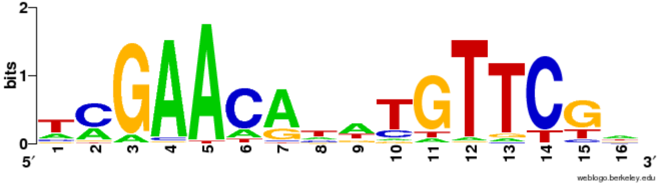
...

Applications of Uncertain Strings

Bioinformatics

- introduced in:

  Stormo, Schneider, Gold, and Ehrenfeucht (1982). "Use of the 'Perceptron' algorithm to distinguish translational initiation sites in E. coli". *Nucleic Acids Research* **10** (9): 2997–3011.
- one of the standard representations of motifs



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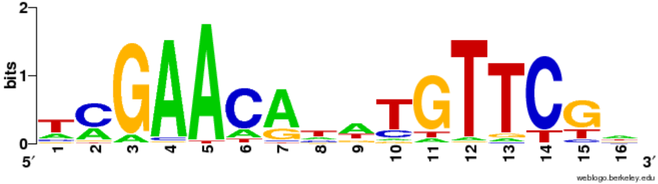

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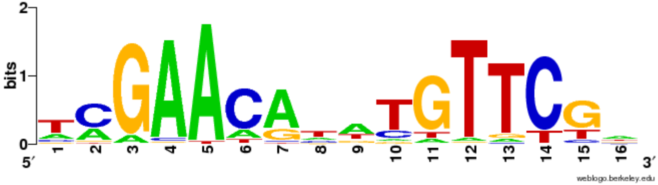

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- Single Nucleotide Polymorphisms, errors in genome sequencing...
- $|\Sigma| = 4$ for DNA sequences
- $|\Sigma| = 20$ for protein sequences

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- Noisy sensor data, Probabilistic databases
  Measurement and sampling errors, resource limitations

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- Noisy sensor data, Probabilistic databases
  Measurement and sampling errors, resource limitations
- Privacy preserving
  Artificial uncertainty can be introduced to sanitize data but keep its utility
- Missing parts of data
  Unknown parameters assumed to take any legal value equally likely

Threshold

- Score function and probability distribution are defined on all solid strings of matching length
- Typically, only high values are considered significant

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Definition

A string S matches a weighed string X if $\mathcal{P}(S, X) \geq \frac{1}{z}$ for a given threshold $\frac{1}{z}$. By $\mathcal{M}_z(X)$ we denote the set of all strings that match X for threshold z.

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Proof. For every $S \in \mathcal{M}_z(X)$, we have $\mathcal{P}(S, X) \geq \frac{1}{z}$. Moreover, $\sum_{S \in \mathcal{M}_z(X)} \mathcal{P}(S, X) \leq 1$.

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- z can be used as a parameter for designing algorithms

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Weighted Pattern Matching

Input

- T – weighted string text of length n ($T[0, n-1]$), represented as an $n \times \sigma$ array
- P – string pattern of length m ($P[0, m-1]$)
- $\frac{1}{z}$ – threshold probability
- Σ – integer alphabet of size σ
- Model: probabilities can be multiplied in $\mathcal{O}(1)$ time

Output

All positions i in T where $\mathcal{P}(P, T[i, i+m-1]) \geq \frac{1}{z}$

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T	a $\frac{1}{4}$ b $\frac{3}{4}$	a 1 b 0	a $\frac{3}{4}$ b $\frac{1}{4}$	a $\frac{1}{2}$ b $\frac{1}{2}$	a 1 b 0
P	a	a	b		

$$z = 8$$

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T	a $\frac{1}{4}$	a 1	a $\frac{3}{4}$	a $\frac{1}{2}$	a 1
	b $\frac{3}{4}$	b 0	b $\frac{1}{4}$	b $\frac{1}{2}$	b 0
P	a	a	b	$\frac{1}{16} < \frac{1}{z}$ **NO**	

$z = 8$

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	b $\frac{3}{4}$	b 0	b $\frac{1}{4}$	b $\frac{1}{2}$	b 0
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	b $\frac{3}{4}$	b 0	b $\frac{1}{4}$	b $\frac{1}{2}$	b 0	
P		a	a	b		$\frac{3}{8} \geq \frac{1}{z}$ **YES**

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P			a	a	b

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P			a	a	b

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	b $\frac{3}{4}$	b 0	b $\frac{1}{4}$	b $\frac{1}{2}$	b 0	
P			a	a	b	$0 < \frac{1}{z}$ **NO**

$z = 8$

Solutions to WPM

$\mathcal{O}(nm)$ time	naïve solution	
$\mathcal{O}(\sigma n \log m)$ time	σ times FFT	[CIMT'04]
$\mathcal{O}(n \log z)$ time	lookahead scoring and k-Mismatch	[Kociumaka-Pissis-R'16]

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WPM via Lookahead Scoring

X	a $\frac{1}{4}$	a 1	a $\frac{3}{4}$	a $\frac{1}{2}$	a 1
	b $\frac{3}{4}$	b 0	b $\frac{1}{4}$	b $\frac{1}{2}$	b 0

WPM via Lookahead Scoring

	X	a $\frac{1}{4}$	a 1	a $\frac{3}{4}$	a $\frac{1}{2}$	a 1
		b $\frac{3}{4}$	b 0	b $\frac{1}{4}$	b $\frac{1}{2}$	b 0
heavy string	**X**	b	a	a	a	a

WPM via Lookahead Scoring

	X	a $\frac{1}{4}$	a 1	a $\frac{3}{4}$	a $\frac{1}{2}$	a 1
		b $\frac{3}{4}$	b 0	b $\frac{1}{4}$	b $\frac{1}{2}$	b 0
heavy string	$\mathbf{X}$	b	a	a	a	a

- $d_H(S, T)$ – Hamming distance between strings S and T (the number of mismatches between S and T)

Fact 2

If $S \in \mathcal{M}_z(X)$ for string S and weighted string X and $\mathbf{X}$ is a heavy string of X, then $d_H(S, \mathbf{X}) \le \log_2 z$.

	X	a $\frac{1}{4}$	a 1	a $\frac{3}{4}$	a $\frac{1}{2}$	a 1
		b $\frac{3}{4}$	b 0	b $\frac{1}{4}$	b $\frac{1}{2}$	b 0
heavy string	$\mathbf{X}$	b	a	a	a	a

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Proof. At each mismatch position between S and $\mathbf{X}$, the probability of the letter of S in X is ≤ 0.5.

WPM via Lookahead Scoring

	X	a $\frac{1}{4}$ b $\frac{3}{4}$	a 1 b 0	a $\frac{3}{4}$ b $\frac{1}{4}$	a $\frac{1}{2}$ b $\frac{1}{2}$	a 1 b 0
heavy string	**X**	b	a	a	a	a

- $d_H(S,T)$ – Hamming distance between strings S and T (the number of mismatches between S and T)

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If $S \in \mathcal{M}_z(X)$ for string S and weighted string X and $\mathbf{X}$ is a heavy string of X, then $d_H(S, \mathbf{X}) \le \log_2 z$.

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- The heavy string method is also known as lookahead scoring

WPM via Lookahead Scoring

k-Mismatch Problem

For two strings P and T, find all positions where P matches T with at most k mismatches (and recover the mismatches).

WPM via Lookahead Scoring

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k-Mismatch can be solved in $\mathcal{O}(nk)$ time using kangaroo jumps:

1. Construct a data structure for answering *lcp*-queries for $T\#P$ ($\mathcal{O}(n+m)$ time via SA and RMQ)
2. For every position i in T, ask at most $k+1$ *lcp*-queries:

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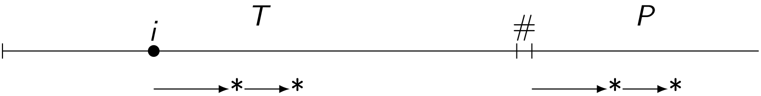

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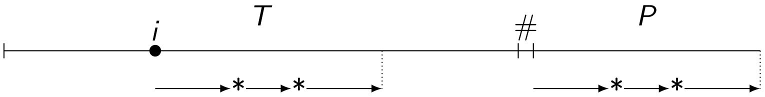

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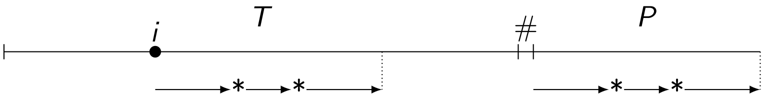

Break after reaching the end of the string or after the $(k+1)$th mismatch

WPM via Lookahead Scoring

1. Compute $\alpha := \mathcal{P}(\mathbf{T}[0, m-1], T[0, m-1])$
2. $k := \log_2 z$
3. For every position $i := 0$ to $n - m$ do:
   1. If $\mathbf{T}[i, i+m-1]$ and P have at most k mismatches, let A be the set of their positions:
      1. $\alpha' := \alpha$
      2. For every $j \in A$,
         $\alpha' := \alpha' \cdot \mathcal{P}(\mathbf{T}[i+j], T[i+j]) / \mathcal{P}(P[j], T[i+j])$
      3. If $\alpha' \geq \frac{1}{z}$, return a match at position i
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		b $\frac{3}{4}$	b 0	b $\frac{1}{4}$	b $\frac{1}{2}$	b 0
heavy string	$\mathbf{T}$	b	a	a	a	a

$z = 8$

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	T	a $\frac{1}{4}$	a 1	a $\frac{3}{4}$	a $\frac{1}{2}$	a 1
		b $\frac{3}{4}$	b 0	b $\frac{1}{4}$	b $\frac{1}{2}$	b 0
heavy string	$\mathbf{T}$	b	a	a	a	a
	$z = 8$	a	a	b		

WPM via Lookahead Scoring

1. Compute $\alpha := \mathcal{P}(\mathbf{T}[0, m-1], T[0, m-1])$
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	T	a $\frac{1}{4}$	a 1	a $\frac{3}{4}$	a $\frac{1}{2}$	a 1
		b $\frac{3}{4}$	b 0	b $\frac{1}{4}$	b $\frac{1}{2}$	b 0
heavy string	$\mathbf{T}$	b	a	a	$\frac{9}{16}$	
	$z = 8$	a	a	b		

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		b $\frac{3}{4}$	b 0	b $\frac{1}{4}$	b $\frac{1}{2}$	b 0
heavy string	$\mathbf{T}$	b	a	a	$\frac{9}{16}$	
		$\neq$	$=$	$\neq$		
$z = 8$		a	a	b		

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heavy string	$\mathbf{T}$	b	a	a	$\frac{9}{16}$	
		$\nparallel$	$\parallel$	$\nparallel$		
$z = 8$		a	a	b	$\frac{1}{16} < \frac{1}{z}$ **NO**	

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	T	a $\frac{1}{4}$ b $\frac{3}{4}$	a 1 b 0	a $\frac{3}{4}$ b $\frac{1}{4}$	a $\frac{1}{2}$ b $\frac{1}{2}$	a 1 b 0
heavy string	$\mathbf{T}$	b	a	a	$\frac{9}{16}$	
	$z = 8$		a	a	b	

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heavy string	$\mathbf{T}$		a	a	a	$\frac{3}{8}$
$z = 8$			a	a	b	

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heavy string	$\mathbf{T}$		a	a	a	$\frac{3}{8}$
			$\parallel$	$\parallel$	$\nparallel$	
	$z = 8$		a	a	b	

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heavy string $\mathbf{T}$		a	a	a	$\frac{3}{8}$
		$\vert\vert$	$\vert\vert$	$\not\vert\vert$	
$z = 8$		a	a	b	$\frac{3}{8} \geq \frac{1}{z}$ **YES**

Application: Profile Matching

Input

- T – string text of length n
- P – pattern being an $m \times \sigma$ profile
- Z – threshold
- σ – alphabet size

Output

All positions i in T where $T[i, i+m-1]$ matches P with score at least Z

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$\mathcal{O}(nm)$ time	naïve solution with heuristics	see [Pizzi-Ukkonen'08]
$\mathcal{O}(\sigma n \log m)$ time	σ times FFT	[Rajasekaran-Jin-Spouge'02]
$\mathcal{O}(n \log \lvert\mathcal{M}_Z(P)\rvert)$ time	lookahead scoring and k-Mismatch	[Kociumaka-Pissis-R'16]

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If a string S matches a profile P with score at least Z and $d_H(S, \mathbf{P}) = k$, then $|\mathcal{M}_Z(P)| \geq 2^k$.

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Hence, $k \leq \log |\mathcal{M}_Z(P)|$.

Lookahead scoring:

- Matching heavy string $\mathbf{P}$ in the text T allowing mismatches – kangaroo jumps
- Start at next position if the score drops below Z

Plan of Presentation

1. Weighted Strings
2. Weighted Pattern Matching and Profile Matching
3. General Weighted Pattern Matching
4. Weighted Indexing
5. On-line and Streaming Weighted Pattern Matching
6. Weighted LCS and SCS

Three variants of WPM

SPWT (WPM)

Input: T – **weighted** string text, P – string pattern
Output:
All positions i in T where $\mathcal{P}(P, T[i, i+m-1]) \geq \frac{1}{z}$

WPST

Input: T – string text, P – **weighted** string pattern
Output:
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WPWT (General WPM)

Input: T – **weighted** string text, P – **weighted** string pattern
Output:
All positions i in T for which there exists a string S such that $\mathcal{P}(S, P) \geq \frac{1}{z}$ and $\mathcal{P}(S, T[i, i+m-1]) \geq \frac{1}{z}$

Three variants of WPM

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Input: T – **weighted** string text, P – string pattern
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WPST

Input: T – string text, P – **weighted** string pattern
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Off-line solutions for WPST and SPWT are the same

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General WPM

T	a $\frac{1}{8}$	a $\frac{1}{2}$	a $\frac{1}{2}$	a 1	a 0	a $\frac{1}{2}$	a $\frac{1}{2}$	$\frac{1}{8}$
	b $\frac{7}{8}$	b $\frac{1}{2}$	b $\frac{1}{2}$	b 0	b 1	b $\frac{1}{2}$	b $\frac{1}{2}$	
P		a $\frac{1}{4}$	a 1	a $\frac{3}{4}$	a $\frac{1}{2}$	a 1		$\frac{9}{32}$
		b $\frac{3}{4}$	b 0	b $\frac{1}{4}$	b $\frac{1}{2}$	b 0		

$$z = 8$$

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T	a $\frac{1}{8}$	a $\frac{1}{2}$	a $\frac{1}{2}$	a 1	a 0	a $\frac{1}{2}$	a $\frac{1}{2}$	$\frac{1}{8}$
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P		a $\frac{1}{4}$	a 1	a $\frac{3}{4}$	a $\frac{1}{2}$	a 1		$\frac{9}{32}$
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- λ – the maximum number of letters with probability $\geq \frac{1}{z}$ at one position ($\lambda \leq \min(z, \sigma)$)

$\mathcal{O}(nz^2 \log z)$ time	[Barton-Liu-Pissis'15]
$\mathcal{O}(n\sqrt{z\lambda}(\log \log z + \log \lambda))$ time	[Kociumaka-Pissis-R'16]

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$\mathcal{O}(n\sqrt{z}\log^2 z)$ time for $\sigma = \mathcal{O}(1)$	this presentation

Weighted Consensus

- Weighted Consensus Problem (WCP): $n = m$
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Moreover, there is a string $S \in \mathcal{M}_z(X) \cap \mathcal{M}_z(Y)$ such that $S[i] = \mathbf{X}[i] = \mathbf{Y}[i]$ unless $\mathbf{X}[i] \neq \mathbf{Y}[i]$.

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- Using k-Mismatch for $k = 2\log_2 z$, in $\mathcal{O}(n \log z)$ time General WPM reduces to $n - m + 1$ instances of WCP of size $\mathcal{O}(\log z)$

Weighted Consensus

Fact 4

If $S \in \mathcal{M}_z(X)$ for a string S and weighted sequence X of length n, then there exists a position i such that $\mathcal{P}(S[0, i-1], X[0, i-1])$, $\mathcal{P}(S[i+1, n-1], X[i+1, n-1]) \geq \frac{1}{\sqrt{z}}$.

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S

X

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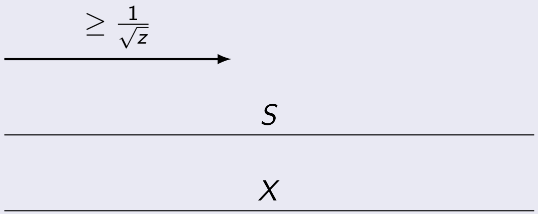

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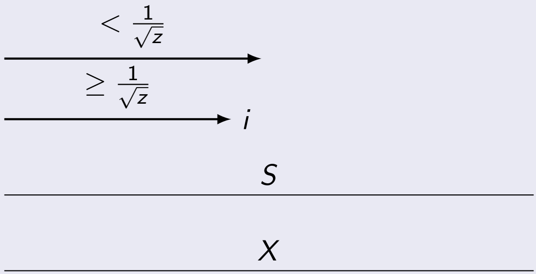

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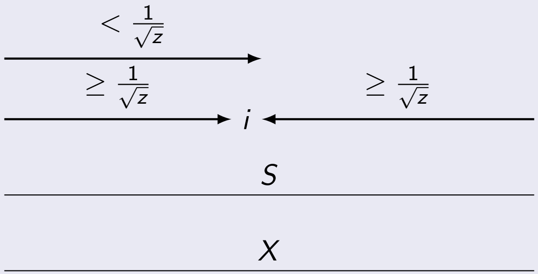

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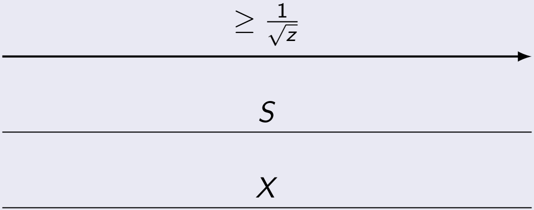

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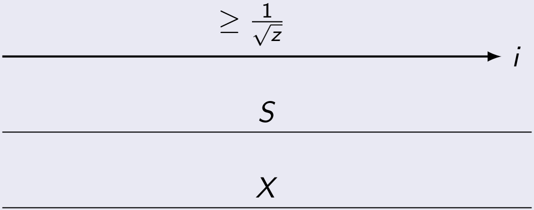

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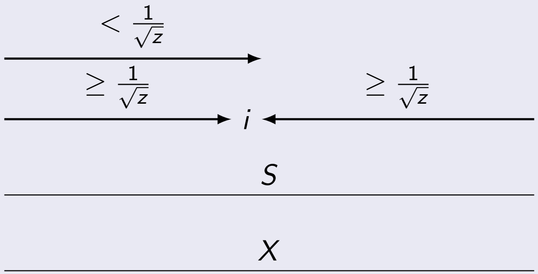

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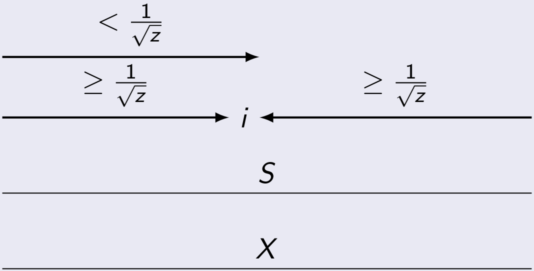

- By Fact 1, $|\mathcal{M}_{\sqrt{z}}(X)| \leq \sqrt{z}$ for a weighted sequence X

Weighted Consensus

Meet-in-the-middle:

1. For $i = 0, \ldots, n-1$ in order, compute $\mathcal{M}_{\sqrt{z}}(T[0, i])$
2. For $i = n, \ldots, 0$ in order, compute $\mathcal{M}_{\sqrt{z}}(T[i, n-1])$
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In total: $\mathcal{O}(\sqrt{z} \log^2 z)$ time.

General WPM and WCP

$\mathcal{O}(nz^2 \log z)$ time	[Barton-Liu-Pissis'15]
$\mathcal{O}(n\sqrt{z\lambda}(\log\log z + \log\lambda))$ time	[Kociumaka-Pissis-R'16]
$\mathcal{O}(n\sqrt{z}\log^2 z)$ time for $\sigma = \mathcal{O}(1)$	this presentation

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Lower bounds[1]; see [Kociumaka-Pissis-R'16]:

no $\mathcal{O}^*(z^{\varepsilon})$ time for every $\varepsilon > 0$	unless the Exponential Time Hypothesis fails
no $\mathcal{O}^*(z^{0.5-\varepsilon})$ time for some $\varepsilon > 0$	unless a better algorithm for Subset Sum
no $\tilde{\mathcal{O}}(z^{0.5}\lambda^{0.5-\varepsilon})$ time for some $\varepsilon > 0$ and $n = \mathcal{O}(1)$	unless 3-Sum conjecture fails

[1] $\mathcal{O}^*$ and $\tilde{\mathcal{O}}$ suppress polynomial and polylog factors with respect to the instance size, resp.

Subset Sum and 3-Sum

Problem definitions:

- Subset Sum
  Input: a set A of n integers
  Output: a subset $B \subseteq A$ summing up to a given integer q, if any
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  Input: three sets A, B, C of λ integers each
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Conditional hardness:

- No $\mathcal{O}(2^{0.5n-\varepsilon})$-time solution for Subset Sum is known ($\varepsilon > 0$)
- No $\mathcal{O}(\lambda^{2-\varepsilon})$-time solution for 3-Sum is known for $\varepsilon > 0$ (3-Sum conjecture)

Efficient Average-Case Algorithms for WPM

problem	preprocessing	avg search time	
WPST	$\mathcal{O}(m\sigma)$	$o(n)$ for small enough z/m	[Barton-Liu-Pissis'18]
SPWT	$\mathcal{O}(m)$	$\mathcal{O}(\frac{nz \log m}{m})$	[Barton-Liu-Pissis'16]
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Implementations provided

Plan of Presentation

1. Weighted Strings
2. Weighted Pattern Matching and Profile Matching
3. General Weighted Pattern Matching
4. Weighted Indexing
5. On-line and Streaming Weighted Pattern Matching
6. Weighted LCS and SCS

Weighted Indexing

Input

- T – weighted string text of length n
- $\frac{1}{z}$ – threshold probability
- $\sigma = \mathcal{O}(1)$ – alphabet size (this presentation)

Query

- Input: P – string pattern of length m
- Output: $\mathrm{Occ}_z(P, T)$ – the set of occurrences of P in T

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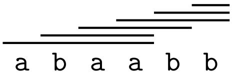

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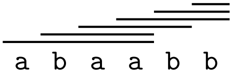

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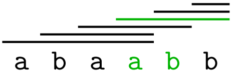

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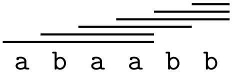

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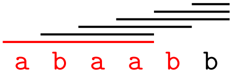

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Input: a weighted sequence X of length n and a threshold z

Output: one can construct a family $\mathcal{S} = (S_j, \pi_j)$ of $\lfloor z \rfloor$ strings of length n with properties such that, for a string P and position i,

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a b a a b b

a a a a a a

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$z = 4$

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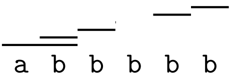

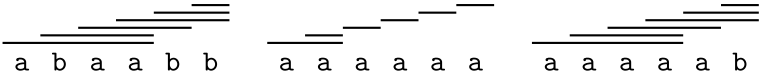

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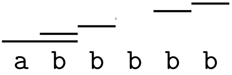

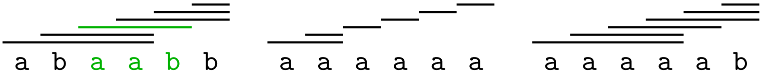

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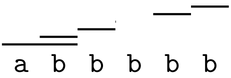

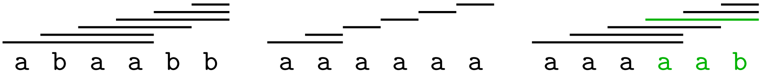

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a a a a a b

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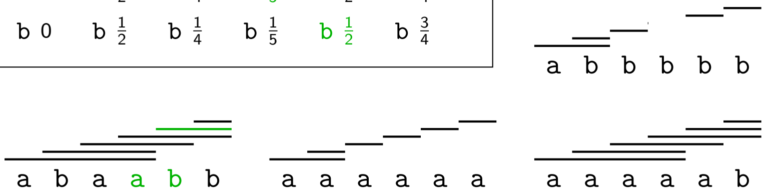

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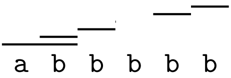
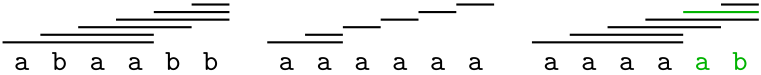
b 0	b $\frac{1}{2}$	b $\frac{1}{4}$	b $\frac{1}{5}$	b $\frac{1}{2}$	b $\frac{3}{4}$





$z = 4,\ P = \mathtt{ab},\ i = \mathbf{4},\ prob = 0.375$

Property Indexing

Input:

- S – string of length n
- π – property
- $\sigma = \mathcal{O}(1)$ – alphabet size (this presentation)

Query:

- Input: P – string pattern of length m
- Output: $\mathrm{Occ}_{\pi}(P, S)$

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space	**construction**	
$\mathcal{O}(n)$	$\mathcal{O}(n \log\log n)$	[ACIKZ'06]
$\mathcal{O}(n)$	$\mathcal{O}(n)$	[Iliopoulos-Rahman'08], [Juan-Liu-Wang'09]

Query time: $\mathcal{O}(m + |\mathrm{Occ}_\pi(P, S)|)$

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- In [BKLPR'16]: a simpler construction based on Ukkonen's algorithm

Weighted Indexing

1. Construct a special family $\mathcal{S}$ of strings with properties ($\mathcal{O}(nz)$ time)
2. Concatenate the strings from $\mathcal{S}$ into a string S with property π
3. Construct a data structure for Property Indexing ($\mathcal{O}(nz)$ time and space, $\mathcal{O}(m + |\mathrm{Occ}_\pi(P, S)|)$ queries)

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Solution: Use colored range listing of [Muthukrishnan'02].

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- The index answers decision and counting queries in $\mathcal{O}(1)$ time (for counting, we use the Color Set Size problem of [Hui'92])

Weighted Indexing

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Solution: Use colored range listing of [Muthukrishnan'02].

- The index answers decision and counting queries in $\mathcal{O}(1)$ time (for counting, we use the Color Set Size problem of [Hui'92])
- And we provide its implementation

Approximate Weighted Indexing

Input

- T – weighted string text of length n
- $\varepsilon > 0$ – allowed error
- z – maximum threshold probability (optional)

Query

- Input: P – string pattern of length m, $z' \le z$ – threshold probability
- Output: Occ – the set of occurrences of P in T with probab. $\ge \frac{1}{z'}$, allowing occurrences with probab. $\ge \frac{1}{z'} - \varepsilon$

Approximate and Generalized Weighted Indexing

Approximate Weighted Indexing

Input

- T – weighted string text of length n
- $\varepsilon > 0$ – allowed error
- z – maximum threshold probability (optional)

Query

- Input: P – string pattern of length m, $z' \leq z$ – threshold probability
- Output: Occ – the set of occurrences of P in T with probab. $\geq \frac{1}{z'}$, allowing occurrences with probab. $\geq \frac{1}{z'} - \varepsilon$

space	construction	query	
$\mathcal{O}(\frac{n}{\varepsilon}z^2)$	$\Omega(\frac{1}{\varepsilon}n^2z^2)$	$\mathcal{O}(m + \lvert\mathrm{Occ}\rvert)$	[BPTS'16]
$\mathcal{O}(\frac{n}{\varepsilon})$	$\mathcal{O}(\frac{n}{\varepsilon}\log\frac{n}{\varepsilon})$	$\mathcal{O}(m + \lvert\mathrm{Occ}\rvert)$	[BKLPR'16]

Approximate and Generalized Weighted Indexing

Generalized Weighted Indexing

Input

- T – weighted string text of length n
- z – maximum threshold probability

Query

- Input: P – string pattern of length m, $z' \leq z$ – threshold probability
- Output: Occ – the set $\mathrm{Occ}_{z'}(P, T)$

Approximate and Generalized Weighted Indexing

Generalized Weighted Indexing

Input

- T – weighted string text of length n
- z – maximum threshold probability

Query

- Input: P – string pattern of length m, $z' \leq z$ – threshold probability
- Output: Occ – the set $\mathrm{Occ}_{z'}(P, T)$

space	**query**	
$\mathcal{O}(\frac{n}{\varepsilon} z^2 \log z)$	$\mathcal{O}(m + m\,\lvert\mathrm{Occ}\rvert)$	[BPTS'16]
$\mathcal{O}(\frac{n}{\varepsilon})$	$\mathcal{O}(m + m\,\lvert\mathrm{Occ}\rvert)$	[BKLPR'16]

Approximate and Generalized Weighted Indexing

Fact 5′

Input: a weighted sequence X of length n and a threshold z

Output: one can construct a family $\mathcal{S} = (S_j, \pi_j)$ of $\lfloor z \rfloor$ strings of length n with properties such that, for a string P and position i,

$$\lfloor z\,\mathcal{P}(P, T[i, i+m-1]) \rfloor \;=\; |\{j \,:\, i \in \mathrm{Occ}_\pi(P, S_j)\}|.$$

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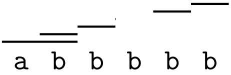
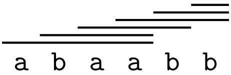
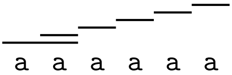
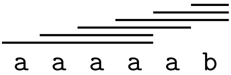
a 1	a $\frac{1}{2}$	a $\frac{3}{4}$	a $\frac{4}{5}$	a $\frac{1}{2}$	a $\frac{1}{4}$
b 0	b $\frac{1}{2}$	b $\frac{1}{4}$	b $\frac{1}{5}$	b $\frac{1}{2}$	b $\frac{3}{4}$









$z = 4$

Approximate and Generalized Weighted Indexing

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Input: a weighted sequence X of length n and a threshold z

Output: one can construct a family $\mathcal{S} = (S_j, \pi_j)$ of $\lfloor z \rfloor$ strings of length n with properties such that, for a string P and position i,

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a 1	a $\frac{1}{2}$	a $\frac{3}{4}$	a $\frac{4}{5}$	a $\frac{1}{2}$	a $\frac{1}{4}$
b 0	b $\frac{1}{2}$	b $\frac{1}{4}$	b $\frac{1}{5}$	b $\frac{1}{2}$	b $\frac{3}{4}$

a b b b b b

a b a a b b

a a a a a a

a a a a a b

$z = 4,\ P = \mathrm{a},\ i = 0,\ prob = 1$

Fact 5′

Input: a weighted sequence X of length n and a threshold z

Output: one can construct a family $\mathcal{S} = (S_j, \pi_j)$ of $\lfloor z \rfloor$ strings of length n with properties such that, for a string P and position i,

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a 1	a $\frac{1}{2}$	a $\frac{3}{4}$	a $\frac{4}{5}$	a $\frac{1}{2}$	a $\frac{1}{4}$
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a b b b b b

a b a a b b   a a a a a a   a a a a a b

$z = 4,\ P = \mathrm{ab},\ i = 0,\ prob = 0.5$

Approximate and Generalized Weighted Indexing

Fact 5′

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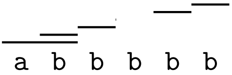

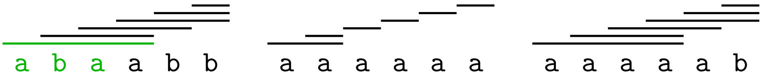

$z = 4,\ P = \text{aba},\ i = 0,\ prob = 0.375$

Plan of Presentation

1. Weighted Strings
2. Weighted Pattern Matching and Profile Matching
3. General Weighted Pattern Matching
4. Weighted Indexing
5. On-line and Streaming Weighted Pattern Matching
6. Weighted LCS and SCS

On-line WPM

Input:

- a pattern P of length m

Output:

- after $T[i]$ has been read, say if P matches $T[i-m+1,i]$

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Objective:

- minimize time of processing a text position
- minimize working space

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- We consider WPST, SPWT, WPWT

On-line WPM

Input:

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Output:

- after $T[i]$ has been read, say if P matches $T[i-m+1,i]$

Objective:

- minimize time of processing a text position
- minimize working space

- We consider WPST, SPWT, WPWT
- All the previous algorithms were *not* on-line (FFT; suffix array of $T\$P$; meet-in-the-middle)

On-line WPM

Black-box scheme [Clifford-Efremenko-Porat-Porat'08]

Pattern matching off-line $\rightarrow$ on-line

On-line WPM

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Pattern matching off-line $\rightarrow$ on-line

Off-line: $T(n, m)$ total time and $S(n, m)$ space
On-line: $\frac{1}{n}T(n, m)\log m$ time per position and $S(m, m)$ space

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Space to store a weighted string of length m: $\mathcal{O}(m \min(\sigma, z))$

problem	time/position	space	
WPST	$\mathcal{O}(\sigma \log^2 m)$	pattern	FFT+Scheme
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WPST	$\mathcal{O}(\sigma\log^2 m)$	$\mathcal{O}(m+z)$	[CIPR'19]
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SPWT	$\mathcal{O}(z + \sigma)$	$\mathcal{O}(m + z)$	[CIPR'19]

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Black-box scheme [Clifford-Efremenko-Porat-Porat'08]

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SPWT	$\mathcal{O}(z+\sigma)$	$\mathcal{O}(m+z)$	[CIPR'19]
WPWT	$\mathcal{O}(z+\sigma)$	$\mathcal{O}(mz^2)$	[CIPR'19]

Streaming WPM

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Streaming pattern matching on strings

First read the pattern, then read the text reporting the occurrences

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Streaming pattern matching on strings

First read the pattern, then read the text reporting the occurrences

space	time/position	
$\mathcal{O}(\log m)$	$\mathcal{O}(\log m)$ whp.	[Porat-Porat'09]
$\mathcal{O}(\log m)$	$\mathcal{O}(1)$ whp.	[Breslauer-Galil'11]

Streaming WPM

- In the Lookahead scoring for SPWT, we considered all occurrences of the pattern P in the heavy string **T** with at most $\log_2 z$ mismatches

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Streaming k-Mismatch on strings

"Error Correcting" (EC): Reports the positions of mismatches and differences of letters at these positions

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Streaming k-Mismatch on strings

"Error Correcting" (EC): Reports the positions of mismatches and differences of letters at these positions

space	time/position	EC	
$\tilde{O}(k^3)$	$\tilde{O}(k^2)$	No	[Porat-Porat'09]
$\tilde{O}(k^2)$	$\tilde{O}(\sqrt{k})$	No	[CFPSS'16]
$\tilde{O}(k^2)$	$\tilde{O}(k)$	Yes	[R-Starikovskaya'17]

Streaming WPM

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$\tilde{\mathcal{O}}(k^2)$	$\tilde{\mathcal{O}}(k)$	Yes	[R-Starikovskaya'17]
$\tilde{\mathcal{O}}(k)$	$\tilde{\mathcal{O}}(k)$	Yes	[Clifford-Kociumaka-Porat'19]

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$\tilde{\mathcal{O}}(k^2)$	$\tilde{\mathcal{O}}(k)$	Yes	[R-Starikovskaya'17]
$\tilde{\mathcal{O}}(k)$	$\tilde{\mathcal{O}}(k)$	Yes	[Clifford-Kociumaka-Porat'19]

S_k and T_k – space and time/position for streaming k-Mismatch

Streaming WPM

In [R-Starikovskaya'17] using Streaming k-Mismatch:

problem	**space**	**time/position**	**approx**
WPST	$\mathcal{O}(z + S_{\log z})$	$\mathcal{O}(\log^2 z + T_{\log z})$	
SPWT	$\mathcal{O}(z \log_{\frac{1}{1-\varepsilon}} z + S_{\log z})$	$\mathcal{O}(z \log_{\frac{1}{1-\varepsilon}} z + T_{\log z})$	$1-\varepsilon$

Streaming WPM

In [R-Starikovskaya'17] using Streaming k-Mismatch:

problem	**space**	**time/position**	**approx**
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SPWT	$\mathcal{O}(z \log_{\frac{1}{1-\varepsilon}} z + S_{\log z})$	$\mathcal{O}(z \log_{\frac{1}{1-\varepsilon}} z + T_{\log z})$	$1-\varepsilon$

In [R-Starikovskaya'17] using Streaming MultiPattern Matching:

problem	**space**	**time/position**	**appr.**
WPST	$\mathcal{O}(z \log m)$	$\mathcal{O}(1)$	
SPWT	$\mathcal{O}(z(\log_{\frac{1}{1-\varepsilon}} z + \log m))$	$\mathcal{O}(z \log_{\frac{1}{1-\varepsilon}} z)$	$1-\varepsilon$
WPWT	$\mathcal{O}(z(\log_{\frac{1}{1-\varepsilon}} z + \log z \log m))$	$\mathcal{O}(z(\log_{\frac{1}{1-\varepsilon}} z + \log z \log m))$	$1-\varepsilon$

Streaming WPM

In [R-Starikovskaya'17] using Streaming k-Mismatch:

problem	space	time/position	approx
WPST	$\mathcal{O}(z + S_{\log z})$	$\mathcal{O}(\log^2 z + T_{\log z})$	
SPWT	$\mathcal{O}(z \log_{\frac{1}{1-\varepsilon}} z + S_{\log z})$	$\mathcal{O}(z \log_{\frac{1}{1-\varepsilon}} z + T_{\log z})$	$1-\varepsilon$

In [R-Starikovskaya'17] using Streaming MultiPattern Matching:

problem	space	time/position	appr.
WPST	$\mathcal{O}(z \log m)$	$\mathcal{O}(1)$	
SPWT	$\mathcal{O}(z(\log_{\frac{1}{1-\varepsilon}} z + \log m))$	$\mathcal{O}(z \log_{\frac{1}{1-\varepsilon}} z)$	$1-\varepsilon$
WPWT	$\mathcal{O}(z(\log_{\frac{1}{1-\varepsilon}} z + \log z \log m))$	$\mathcal{O}(z(\log_{\frac{1}{1-\varepsilon}} z + \log z \log m))$	$1-\varepsilon$

Lower bound in [R-Starikovskaya'17]

Any streaming algorithm, exact or $(1-\varepsilon)$-approximate, solving WPST, SPWT or WPWT must use $\Omega(z)$ space.

Plan of Presentation

1. Weighted Strings
2. Weighted Pattern Matching and Profile Matching
3. General Weighted Pattern Matching
4. Weighted Indexing
5. On-line and Streaming Weighted Pattern Matching
6. Weighted LCS and SCS

Weighted LCS and SCS

Definition (z-subsequence)

For a string S, a weighted sequence W and a threshold $\frac{1}{z}$, we write $S \subseteq_z W$ if $\mathcal{P}(S, W') \geq \frac{1}{z}$ for some subsequence W' of W.

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Weighted Longest Common Subsequence, [Amir-Gotthilf-Shalom'09]

Input

- W_1, W_2 – weighted strings of length n
- $\frac{1}{z}$ – threshold probability
- $\sigma = \mathcal{O}(1)$ – alphabet size (this presentation)

Output

- A longest string S such that $S \subseteq_z W_1$ and $S \subseteq_z W_2$

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Weighted Shortest Common Supersequence, [Amir-Gotthilf-Shalom'11]

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Weighted LCS and SCS

- Both problems are NP-complete for $\sigma = 2$; see [CKRRW'11] and [CKPRRSWZ'19]

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Weighted SCS

Upper bound

- $\mathcal{O}(n^2\sqrt{z}\log z)$ (using Facts 1, 2, 4); see [CKPRRSWZ'19]

Lower bounds

- $\mathcal{O}(n^{2-\varepsilon})$ unless the Strong Exponential Time Hypothesis fails; see [Abboud-Backurs-Williams'15]
- $\mathcal{O}^*(z^{0.5-\varepsilon})$ unless a better algorithm for Subset Sum exists; see [Kociumaka-Pissis-R]

Weighted LCS and SCS

- Both problems are NP-complete for $\sigma = 2$; see [CKRRW'11] and [CKPRRSWZ'19]

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Weighted LCS

- Cannot be solved in $\mathcal{O}(n\,f(z))$ time or $\mathcal{O}(n^{f(z)})$ time unless $P = NP$; see [CKPRRSWZ'19]

Conclusion

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Open Problems and Further Work

- A Weighted Index with $\mathcal{O}(m + |\mathrm{Occ}_z(P, T)|)$-time queries and $o(nz)$ space?

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- A Weighted Index with $\mathcal{O}(m + |\mathrm{Occ}_z(P, T)|)$-time queries and $o(nz)$ space?
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- More efficient queries in the Generalized Weighed Index?

Open Problems and Further Work

- A Weighted Index with $\mathcal{O}(m + |\mathrm{Occ}_z(P, T)|)$-time queries and $o(nz)$ space?
- An on-line algorithm for General WPM using less space?
- More efficient queries in the Generalized Weighed Index?
- Automatic selection of parameter z?
- Non-independent probability distributions?

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Thank You

Thank you for your attention!