Conversion of Finite Tree Automata to Regular Tree Expressions by State Elimination

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Outline

Problem statement

- Let A be a nondeterministic bottom-up finite tree automaton (FTA).
- Transform A to an equivalent regular tree expression (RTE) E such that $L(E) = L(A)$.
  - Both FTA and RTE describe exactly the class of regular tree languages [CDG+07].
  - The problem is analogous to the (string) conversion from a finite automaton to a regular expression.

Related work

- Converting FTAs to RTEs using regular tree equations (Guellouma, Cherroun [GC18]).

Our approach

- Elimination of states (inspired by well known state elimination algorithm from strings [HMU03]).

Trees

- Trees are one of the fundamental data structures. Useful for hierarchical data (XML, AST, ...).
- Tree is defined by the means of graph theory.
- Our trees are rooted, ordered, labelled, and ranked.
- There is a hierarchy of tree languages, today we deal with *regular tree languages*.

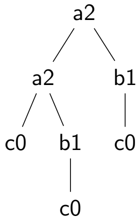


Figure: Tree t over ranked alphabet $\mathcal{A} = \{a2, b1, c0\}$. The number associated with the symbol is the arity of the symbol.

Finite Tree Automata I

- Standard computation model for regular tree languages.

Definition

Non-deterministic bottom-up (also frontier-to-root) finite tree automaton is a quadruple $A = (Q, \Sigma, \Delta, Q_F)$, where

- Q is the set of states,
- Σ is a ranked alphabet (symbols with non-negative arity),
- Δ is a set of transition rules (mapping $\Sigma_n \times Q^n \mapsto \mathcal{P}(Q)$), and
  - $a(q_1, \ldots, q_n) \to q$
- Q_F is the set of final states.

Finite Tree Automata II

- The computation moves from the leaves towards the root.
- Node a with arity n is assigned with state q if $a(q_1, \ldots, q_n) \to q \in \Delta$ and $q_1, \ldots, q_n$ are the states assigned to children of a.
- Tree is accepted by the automaton if the root is assigned with a final state.

Example

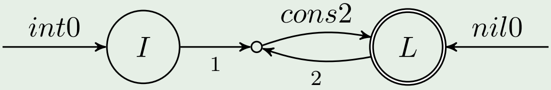
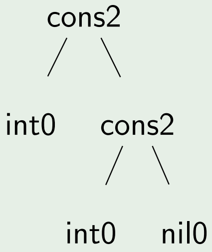


Figure: Example FTA accepting trees representing valid LISP lists consisting of int symbol.



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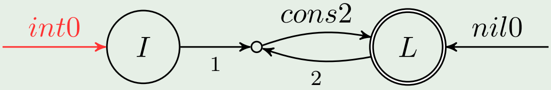

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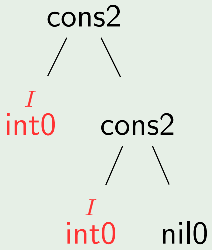

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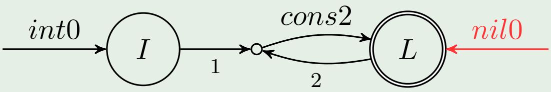

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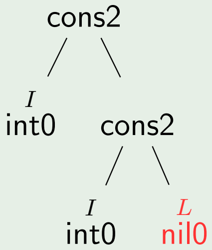

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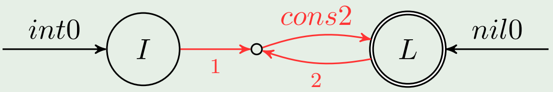

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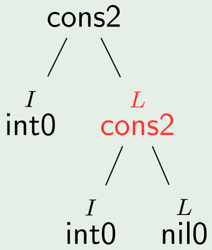

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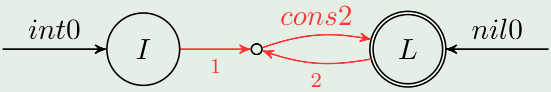

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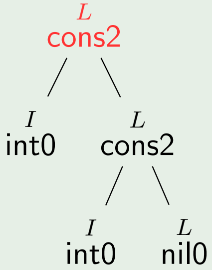


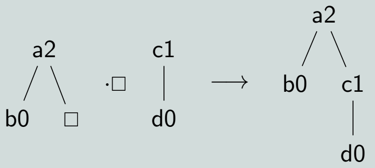
Figure: Run on an example tree accepted by the automaton.

Operations on Trees

Tree substitution

- Substituting occurrences of $\square_i$ by trees from L_i.
- Concatenating trees in specified places.
- $t\,\{\square_1 \leftarrow L_1, \ldots, \square_n \leftarrow L_n\}$

Example



Operations on Tree Languages

- Union: $L_1 + L_2 = L_1 \cup L_2$
- Concatenation: $L_1 \cdot_\square L_2 = \bigcup_{t \in L_1} \{t\,\{\square \leftarrow L_2\}\}$
- Closure: $L^{*,\square} = \bigcup_{n \geq 0} L^{n,\square}$.
  - $L^{0,\square} = \{\square\}, \quad L^{n+1,\square} = L \cdot_\square L^{n,\square}$

Regular Tree Expressions I

- Another way of describing regular tree languages.
- Analogous to regular (string) expressions.
- Defined as in TATA (Comon et al. [CDG$^+$07]): Alphabets of input symbols ($\mathcal{F}$) and substitution symbols ($\mathcal{K}$).

Definition

- $a \in \mathcal{F}_0$ is a RTE
- $\Box \in \mathcal{K}$ is a RTE
- If all E_i are RTEs, $a \in \mathcal{F}$, and $\Box \in \mathcal{K}$, then:
  - $a(E_1, \ldots, E_{arity(a)})$,
  - $E_1 + E_2$,
  - $E_1 \cdot_\Box E_2$, and
  - ${E_1}^{*,\Box}$ are RTEs.

Example

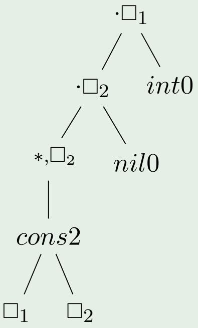
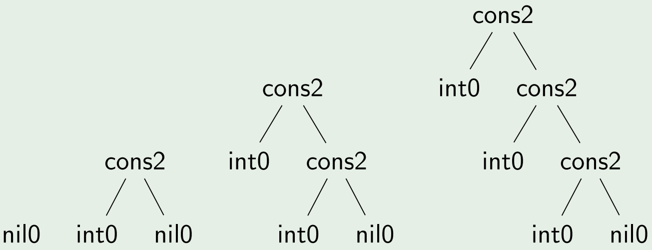


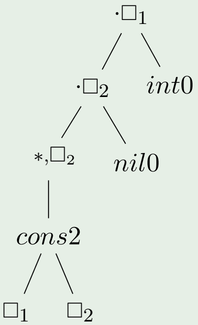
Figure: RTE for valid lists of integers in LISP.

$$\mathcal{F} = \{cons2, int0, nil0\}$$
$$\mathcal{K} = \{\Box_1, \Box_2\}$$



Regular Tree Expressions II

Example



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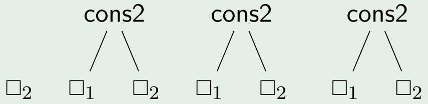

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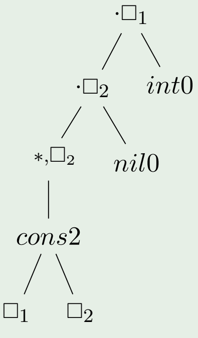

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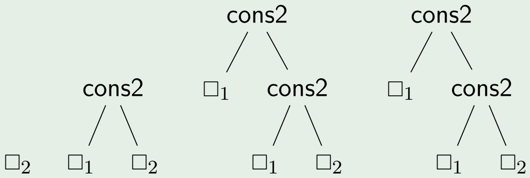

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Regular Tree Expressions II

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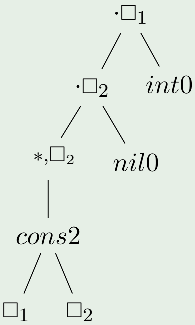

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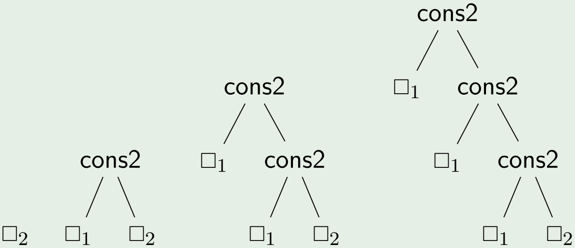

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Regular Tree Expressions II

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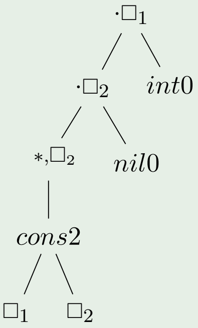

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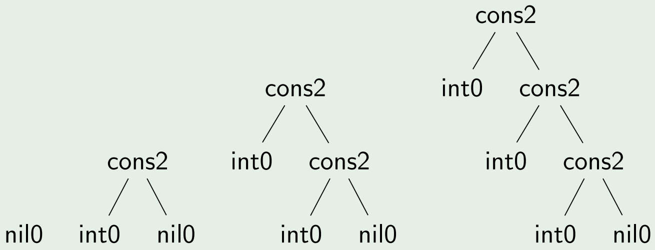


State elimination on FA

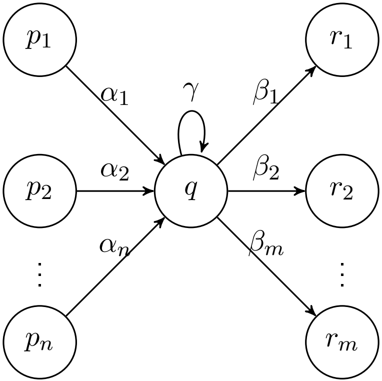


Figure: One step of a state elimination in FA.

- Generalised NFA: transitions use regular expressions instead of symbols
- Eliminate all states except the initial state and final state
  - Replace all paths through q with new transitions

    $$p_i \xrightarrow{\alpha_i \gamma^* \beta_j} r_j$$

  - Remove state q
- The path from the initial state to the final state is the equivalent regular expression.

State elimination on FTA

- Generalised FTA (GFTA): transitions use regular tree expressions instead of symbols.

From FTA to GFTA

- Create RTEs from symbols in the transitions mapping.
  - Source states of the transition will be children of the symbol.
  - Symbols corresponding to states are references to a language of a state.
  - Order of source states of the transition is no longer needed (now defined in the RTE).
- Transform the automaton to have only a one final state (useful later).

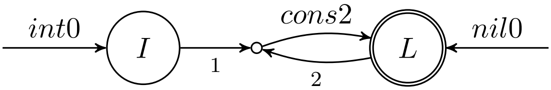


Figure: An example fragment of a FTA.

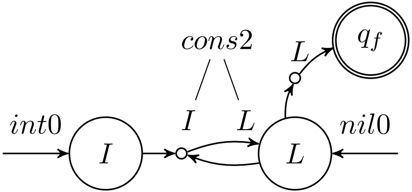


Figure: GFTA corresponding to the FTA on the left.

State elimination on FTA

Transition type

Classification of the GFTA's transitions with respect to a state q:

- incoming (q is a target, but not a source),
- outgoing (q is a source but not a target),
- looping (q is both a source and a target).

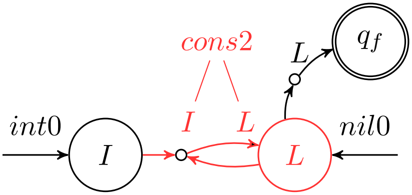


Figure: An example GFTA with a looping transition w.r.t. the state L.

Elimination of a single state from GFTA

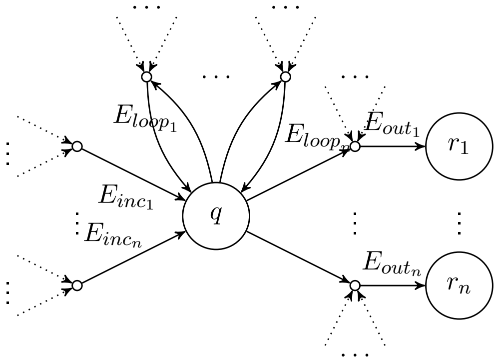
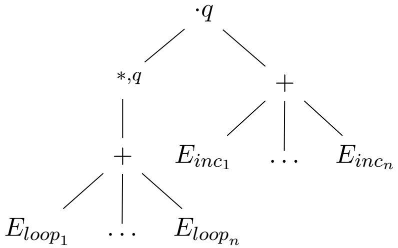


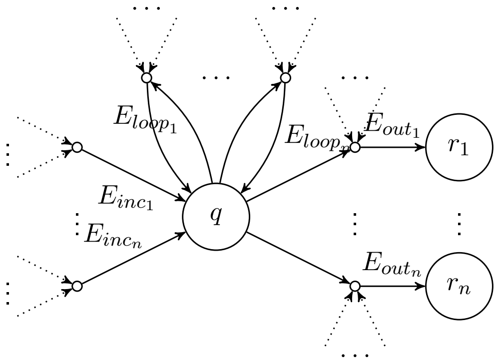
Figure: Situation when eliminating the state q of GFTA.

- E_{out} refers to state q (contains $q \in \mathcal{K}$).
- Language of state q can be seen as RTE with E_{loop} and E_{inc} "subRTEs" [CDG+07].



- Replace q in all E_{out} with this fragment ($O(1)$ if not replacing but using concatenation).

Elimination of a single state from GFTA



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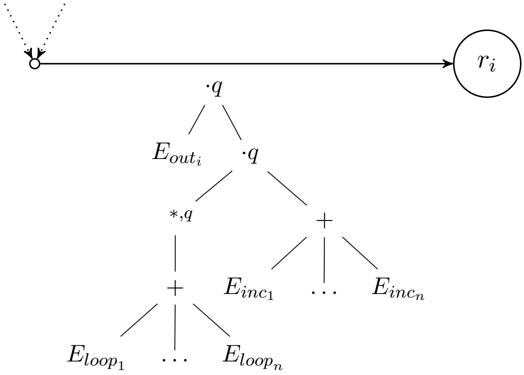


Figure: Modification of an outgoing edge.

Example

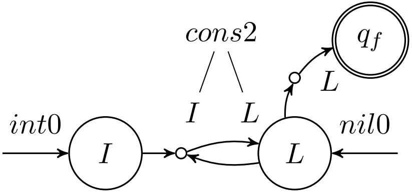


Figure: The original GFTA.

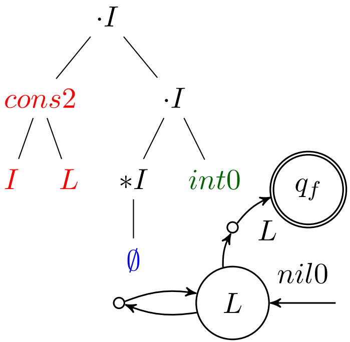
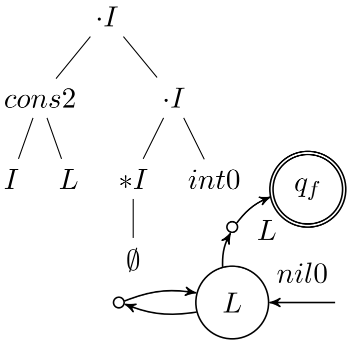


Figure: After eliminating state I.

Example



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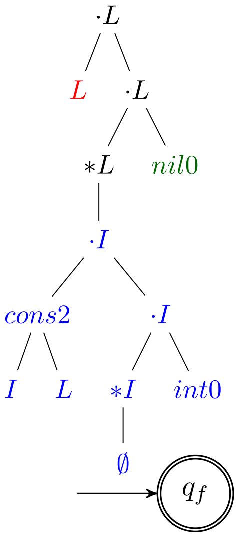

Figure: After eliminating state L.

State elimination on TA

Elimination Algorithm

1. Convert the input FTA to a GFTA.
2. Eliminate any non-final state using the approach on previous slide until a single-state GFTA remains.
3. The resulting RTE can be read from the transitions leading to the final state.

Time Complexity

For an input FTA $A = (Q, \Sigma, \Delta, Q_F)$ and corresponding GFTA $G = (Q \cup \{q_f\}, \Sigma, \Gamma, \{q_f\})$:

Conversion to GFTA is done in $O(|\Delta| + |Q_F|)$.

Eliminating a single state is $O(|Q| \cdot |\Gamma|)$, but both $|Q|$ and $|\Gamma|$ gradually decrease.

Total time consists of conversion and $|Q|$ invocations of elimination step: $O(|Q| \cdot |Q| \cdot (|\Delta| + |Q_F|))$.

Conclusion

- A simple algorithm for the construction of a regular tree expression(RTE) equivalent to given finite tree automaton (FTA) by eliminating states.
- Ideas come from a proof of RTE - FTA equivalence in [CDG$^+$07] and from the similar string algorithm [HMU03].
- Implementation in *Algorithms Library Toolkit* [ALT].

Future work:

- Is there a way of speeding up the elimination step?
- Good and bad elimination orders? Affects the size of the RTE.

5 Appendix: References

References I

- Algorithms Library Toolkit.
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