Forward Linearised Tree Pattern Matching Using Tree Pattern Border Array

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Outline

Trees and Tree Notations

- An unranked alphabet
  $\mathcal{A} = \{a, b, \uparrow\}$
- Subject tree t_{1u} in the prefix bar notation

$pref\_bar(t_{1u}) = a\ a\ a \uparrow a\ a \uparrow\uparrow\uparrow a\ b \uparrow\uparrow\uparrow$

- A ranked alphabet
  $\mathcal{A} = \{a2, a1, a0, b0\}$
- And t_{1r} in the prefix notation

$pref(t_{1r}) = a2\ a2\ a0\ a1\ a0\ a1\ b0$

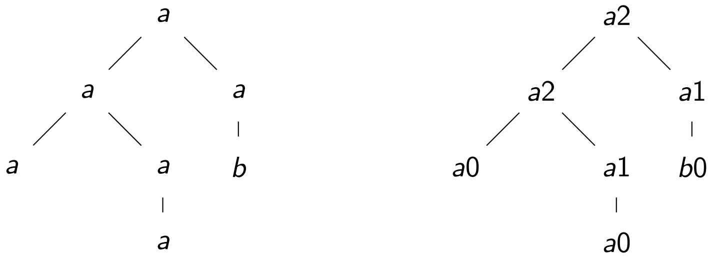


Figure: Subject tree t_{1u} over an unranked alphabet (left), and the same subject tree t_{1r} over a ranked alphabet (right)

Some Other Notations

On a ranked alphabet

- $\mathcal{A} = \{a2, a1, a0, b0\}$, t_{1r} in the postfix notation
  $post(t_{1r}) = a0\ a0\ a1\ a2\ b0\ a1\ a2$
- $\mathcal{A} = \{a2, a1, a0, \uparrow 2, \uparrow 1, \uparrow 0\}$, t_{1r} in the prefix ranked bar notation
  $pref\_ranked\_bar(t_{1r}) = a2\ a2\ a0 \uparrow 0\ a1\ a0 \uparrow 0 \uparrow 1 \uparrow 2\ a1\ b0 \uparrow 0 \uparrow 1 \uparrow 2$
- Euler tour traversal, ...

On an unranked alphabet

- $\mathcal{A} = \{a, (,)\}$, t_{1r} in the prefix bracketed notation
  $pref\_brac(t_{1u}) = a\ (\ a\ (\ a\ (\ )\ a\ (\ a\ (\ )\ )\ )\ a\ (\ b\ (\ )\ )\ )$
- $\mathcal{A} = \{a, \uparrow\}$, t_{1r} in the postfix bar notation
  $post\_bar(t_{1u}) = \uparrow\uparrow\uparrow a \uparrow\uparrow a\ a\ a \uparrow\uparrow b\ a\ a$

Trees Patterns

- A ranked alphabet of a tree template $\mathcal{A} = \{a2, a1, a0, S\}$
- Tree template p_{2r} in the prefix notation $pref(t_{1r}) = a2\ S\ a1\ S$
- Symbol S stands for any subtree.

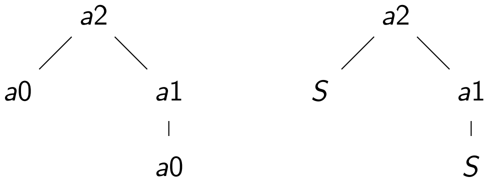


Figure: Subtree p_{1r} (left) of the t_{1r}, tree template p_{2r} (right)

Tree Pattern Matching

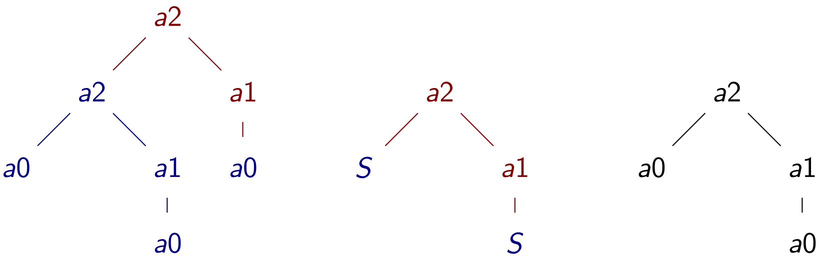
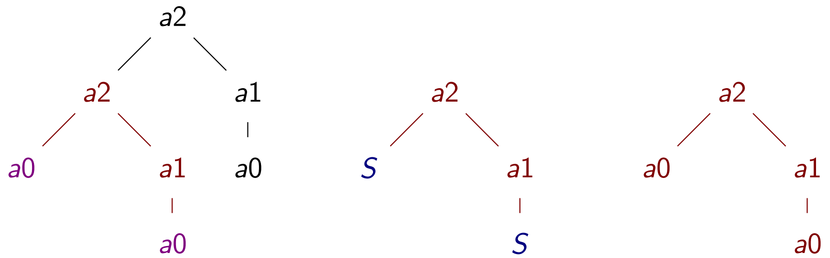


Figure: Tree t_{1r} over a ranked alphabet (left), tree template p_{2r} (centre) and subtree p_{1r} of t_{1r} (right)

- For many linearisations it holds that subtree s of a tree t, the linear respresenation of s is a substring of linear representation of t.

$pref(t_{1r}) = a2\ a2\ a0\ a1\ a0\ a1\ a0$
$pref(p_{1r}) = a2\ a0\ a1\ a0$
$pref(p_{2r}) = a2\ S\ a1\ S$

Tree Pattern Matching



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$pref(p_{2r}) = a2\ S\ a1\ S$

Subtree Jump Table

A structure allowing quick jumps over subtrees of a given linearised tree.

Definition (subtree jump table for prefix notation $sjt(pref(t))$)

Let t and $pref(t) = \ell_1\ell_2 \dots \ell_n$, $n \geq 1$, be a tree and its prefix notation, respectively. A *subtree jump table for prefix notation* $sjt(pref(t))$ is a mapping from a set $\{1..n\}$ into a set $\{2..n+1\}$. If $\ell_i\ell_{i+1} \dots \ell_{j-1}$ is the prefix notation of a subtree of tree t, then $sjt(pref(t))[i] = j$, $1 \leq i < j \leq n+1$.

Table: Subtree jump table $sjt(pref(t_{1r}))$

id	1	2	3	4	5	6	7
$pref(t_{1r})$	$a2$	$a2$	$a0$	$a1$	$a0$	$a1$	$b0$
$sjt(pref(t_{1r}))$	8	6	4	6	6	8	8

Forward Pattern Matching

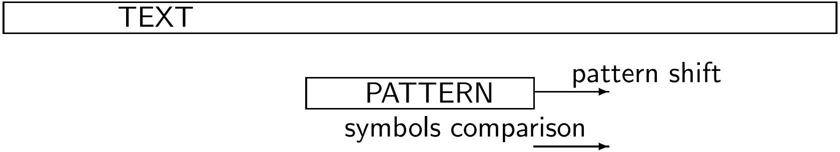


Figure: Graphical outline of a forward pattern matching algorithm

Searching for Subtrees

- A Morris-Pratt algorithm makes use of a border array table.
- A border of a string s a prefix of s that is also a suffix of s.
- The border array stores the length of the longest border for each prefix of s.

An alphabet $\mathcal{A} = \{a2, a0\}$
A string $s_1 = a2\, a0\, a2\, a0\, a0$

Table: The border array for string s_1

1	2	3	4	5
0	0	1	2	0

In order to look for subtrees in a tree, the Morris-Pratt algorithm can be used without changes. Actually, any string pattern matching algorithm can be used.

String Morris-Pratt algorithm

Algorithm 1: Morris-Pratt matching function.

Input: The subject string s of size n, the pattern string p of size m, the border array table $\mathcal{B}(p)$

Result: A list of matches.

```
begin
    i := 0, j := 1
    while i ≤ n − m do
        /* occurrence check loop */
        while j ≤ m and s[i + j] = p[j] do
            j += 1
        end
        if j > m then yield i + 1
        /* shift handling */
        if j ≠ 1 then
            i += j − B(p)[j − 1] − 1 /* j - 1st symbol failed or overflowed */
            j := B(p)[j − 1] + 1
        else
            i += 1
        end
    end
end
```

Alternative border-array definition

Definition (border array $\mathcal{B}(s)$)

Let s be a string of length n. The border array $\mathcal{B}(s)$ is defined for each index $1 \leq i \leq n$ such that $\mathcal{B}(s)[1] = 0$ and otherwise
$\mathcal{B}(s)[i] = max(\{0\} \cup \{k : s[1..k] \;=\; s[i-k+1..i] \wedge k \geq 1 \wedge i-k+1 > 1\})$.

Forward Tree Pattern Matching Algorithm

Modifications to the Morris-Pratt algorithm needed to use it for tree patterns.

- The occurrence check loop has to be modified to handle the wildcards,
- the border array needs to be modified to represent the same idea in tree patterns.

Matches

Definition (matches relation *s matches r*)

Let S be a wildcard symbol representing a complete subtree in prefix ranked notation of trees. Two strings s and r are in relation *matches* if:

$$
\begin{array}{llll}
s = \ell s' & r = \ell r' & \mathit{and} & s' \ \mathit{matches}\ r' \\
 & & \mathit{and} & \ell \in \mathcal{A}, \\
s = Ss' & r = Sr' & \mathit{and} & s' \ \mathit{matches}\ r', \\
s = \ell_1 \ldots \ell_m s' & r = Sr' & \mathit{and} & ac(\ell_1 \ldots \ell_m) = 0 \\
 & & \mathit{and} & \forall k, 1 \leq k < m, ac(\ell_1 \ldots \ell_k) \geq 1 \\
 & & \mathit{and} & s' \ \mathit{matches}\ r', \\
s = Ss' & r = \ell_1 \ldots \ell_m r' & \mathit{and} & ac(\ell_1 \ldots \ell_m) = 0 \\
 & & \mathit{and} & \forall k, 1 \leq k < m, ac(\ell_1 \ldots \ell_k) \geq 1 \\
 & & \mathit{and} & s' \ \mathit{matches}\ r', \\
s = Ss' & r = \ell_1 \ldots \ell_m & \mathit{and} & \forall k, 1 \leq k \leq m, ac(\ell_1 \ldots \ell_k) \geq 1, \\
s = \varepsilon \ \mathit{or}\ r = \varepsilon & & &
\end{array}
$$

Tree pattern border array

Definition (tree pattern border array $\mathcal{B}(pref(p))$)

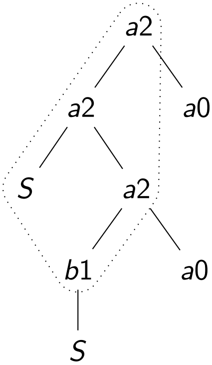
Let $pref(p)$ be a tree pattern in a prefix notation of length n. The $\mathcal{B}(pref(p))$ is defined for each index $1 \leq i \leq n$ such that $\mathcal{B}(pref(p))[1] = 0$ and otherwise $\mathcal{B}(pref(p))[i] = max(\{0\} \cup \{k : pref(p)\ matches\ pref(p)[i-k+1..i] \wedge k \geq 1 \wedge i-k+1 > 1\})$.

Visualization of the Matches Relation

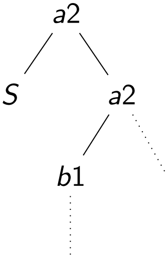
Table: Trace of naive computation of $pref(p)$ $matches$ $pref(p)[j+1..5]$ for $1 \leq j \leq 5$ and $pref(p) = a2\ a2\ S\ a2\ b1\ S\ a0\ a0$.

	1	2	3	4	5	6	7	8			
pref(p)	$a2$	$a2$	S	$a2$	$b1$	S	$a0$	$a0$			
pref(p)[2..5]	$a2$	$\vdash$		S			$\dashv$	$a2$			mismatch at position 8
pref(p)[3..5]	$\vdash$			S				$\dashv$	$a2$	$b1$	match
pref(p)[4..5]	$a2$	$b1$									mismatch at position 2
pref(p)[5..5]	$b1$										mismatch at position 1
pref(p)[6..5]											match (pref(p)[6..5] $= \varepsilon$)

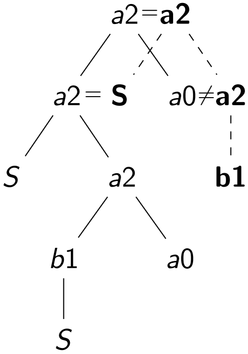
Bad Character Shift Table for Tree Templates



(a) Tree pattern p and the subgraph of p corresponding to the prefix of $pref(p)$ relevant to the computation of relation *matches*.



(b) The subgraph of p corresponding to $pref(p)[2..5]$.



(c) Visualisation of the alignment.

The border array example

Table: The tree pattern border array $\mathcal{B}(pref(p))$ for $pref(p) = a2\ a2\ S\ a2\ b1\ S\ a0\ a0$.

id	1	2	3	4	5	6	7	8
$pref(p)$	$a2$	$a2$	S	$a2$	$b1$	S	$a0$	$a0$
$\mathcal{B}(pref(p))$	0	1	2	2	3	4	5	6

Modification of the Occurrence Check loop

Due to the variable length of the subtree matched to the wildcard

- an offset to the subject is to be maintained,
- the subtree jump table is used to efficiently skip over subtrees.

Algorithm 2: (Fragment) modification of the Occurrence check.

```
offset := i + j
while j ≤ m and offset ≤ n do
    if pref(p)[j] = pref(s)[offset] then
        j += 1
        offset += 1
    else if pref(p)[j] = S then
        offset := sjt(pref(s))[offset]
        j += 1
    else
        break
    end
end
```

Modification of the Shift Handling

Modifications to the shift handling:

- the shift handling is mainly updated by switching to tree pattern border array
- the number of character that do not need to be matched is limited by the distance of the wildcard from the beginning of the pattern.

Algorithm 3: (Fragment) modification of the Shift Handling.

0a $Spos := min(\{k : pref(p)[k] = S \wedge 1 \leq k \leq m\})$
0b $shift[1] := 1$
0c **for** $k := 2$ **to** $m + 1$ **do** $shift[k] := k - \mathcal{B}(pref(p))[k-1] - 1$
/* because the k - 1st symbol failed or overflowed */

$\vdots$

10-15a $i \mathrel{+}= shift(pref(p))[j]$
10-15b $j := max(1, min(Spos, j) - shift(pref(p))[j])$

Sample run of the matching algorithm

Table: The $shift(pref(p))$ for $pref(p) = a2\ a2\ S\ a2\ b1\ S\ a0\ a0$.

id	1	2	3	4	5	6	7	8	9
pref(p)	*a2*	*a2*	*S*	*a2*	*b1*	*S*	*a0*	*a0*	
shift(pref(p))	1	1	1	1	2	2	2	2	2

Table: Run for the subject $pref(s)$ and the pattern $pref(p) = a2\ a2\ S\ a2\ b1\ S\ a0\ a0$.

id	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
pref(s)	*a2*	*a2*	*a2*	*a0*	*a2*	*b1*	*b0*	*a0*	*a0*	*a2*	*a2*	*a0*	*a2*	*b1*	*b0*	*a0*	*a0*
sjt	18	10	9	5	9	8	8	9	10	18	17	13	17	16	16	17	18
1	*a2*	*a2*	$\vdash$			*S*		$\dashv$	*a2*								
2		*a2*	*a2*	*S*	*a2*	*b1*	*S*	*a0*	*a0*								
3				*a2*													
4					*a2*	*a2*											
5						*a2*											
6							*a2*										
7								*a2*									
8									*a2*								
9										*a2*	*a2*	*S*	*a2*	*b1*	*S*	*a0*	*a0*

Complexities

The n is the size of the subject tree, the m is the size of the tree template and the $\mathcal{A}$ is the size of the alphabet.

- The tree pattern border array requires $\Theta(m)$ space.
- The preprocessing (computation of the tree pattern border array) takes $\mathcal{O}(m^2)$ time
- The algorithm runs in $\Omega(n)$ time in the best case and $\mathcal{O}(m \cdot n)$ time in the worst case if searching for tree templates and $\Theta(n)$ time if searching for subtrees.

Measurements

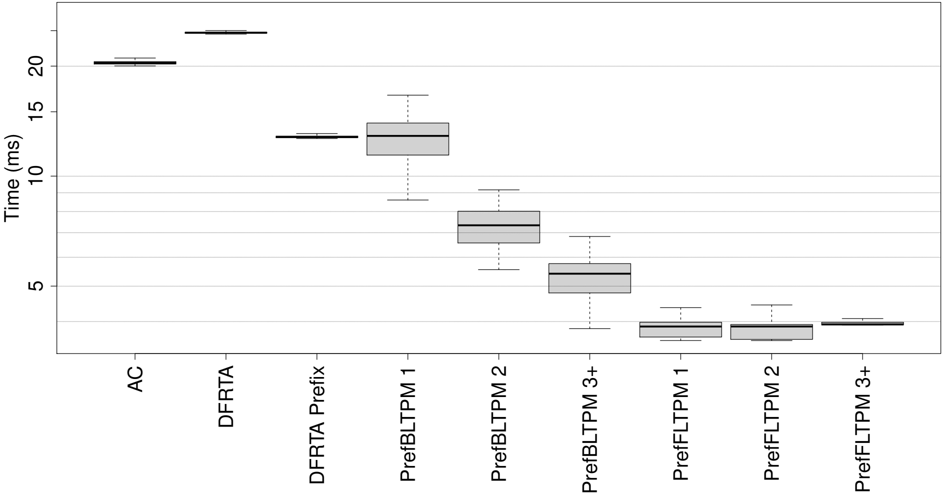
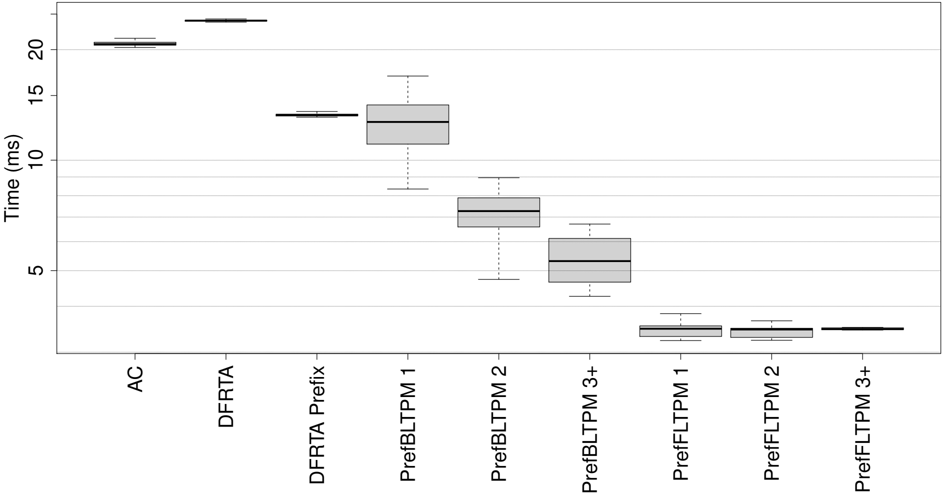


Figure: Results on 150 trees of ca. 500 nodes each with wildcards.

Measurements



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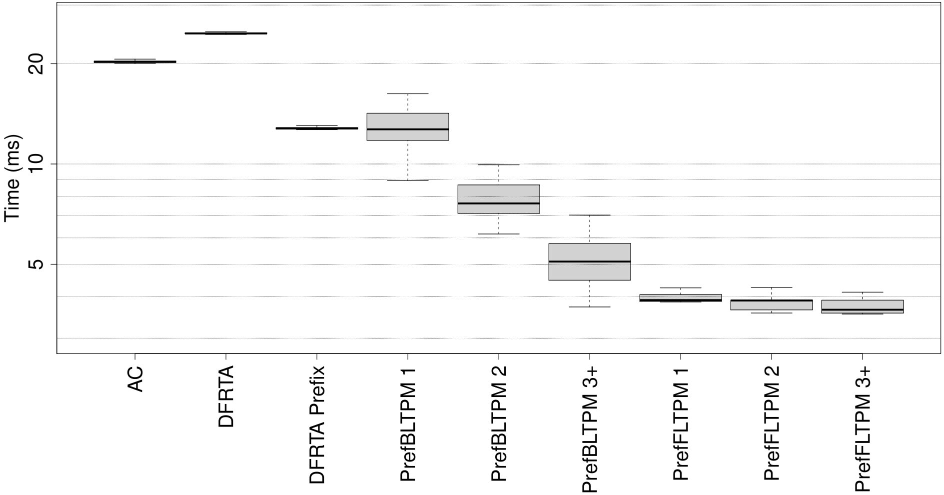


Figure: Results on 150 trees of ca. 500 nodes each without wildcards.

Measurements

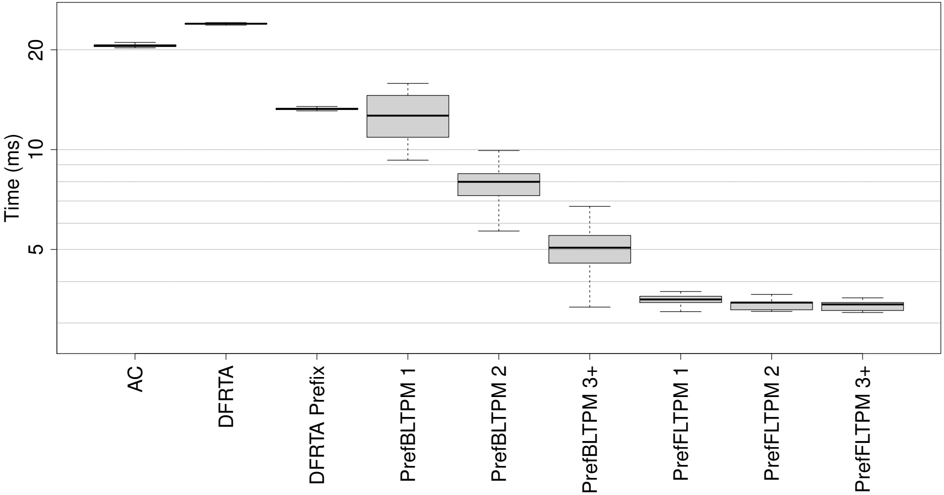


Figure: Results on 500 trees of ca. 150 nodes each without wildcards.

Conclusions

Results:

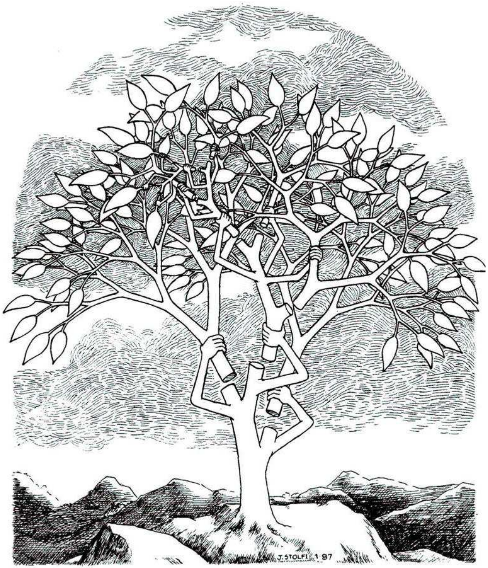
- A new tree pattern matching algorithm was presented,
- the algorithm is based on Morris-Pratt algorithm and uses an adaptation of the border array from string domain,
- the algorithm was implemented in the Forrest-FIRE toolkit and experimentally evaluated using the dataset used in its original evaluation.

Takeaway message:

- Many string processing algorithms can be modified (with some care) to process trees represented as strings using some linearisation schemes.

Future work:

- Adapt the Knuth-Morris-Pratt improvement to the presented algorithm.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