Multi-Grain Relations

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Motivation

Goal

Expressing relations between different representations with two (or more) levels of analysis.

Example

Natural Language Processing, morphological analysis.

[pos=V]	[pos=N][from=V]
move	ment

The two levels: symbols (letters, features), morphemes.

Important operations

The formalism and its compilation

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Important operations

rational operations + Cartesian product + intersection + difference

More about the two grains

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Synchronization between the two representations: using the morpheme notion.

Two-grain Sets

Definition

- $L \in \mathrm{Rat}_1(\Sigma^*)$
- $\mu : \Sigma \longrightarrow \mathrm{Rat}_2(A^* \times B^*)$
- $TGS(L, \mu) = \{(v_1, w_1) \ldots (v_n, w_n) \in (A^* \times B^*)^* |$
  $\exists a_1 \ldots a_n \in L, (v_1, w_1) \in \mu(a_1), \ldots, (v_n, w_n) \in \mu(a_n)\}.$

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Example

- $L = a(a|b)b$
- $\mu(a) = (x^*, x) \;\; \mu(b) = (y, z^+)$
- $$\begin{array}{cccc} a & a & b & \in L \\ (xxx, x) & (xx, x) & (y, z) & \in TGS(L, \mu) \end{array}$$

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- $\begin{matrix} a & a & b & \in L \\ (xxx, x) & (xx, x) & (y, z) & \in TGS(L, \mu) \end{matrix}$

Terminology

- coarse grains: elements from Σ
- fine grains: elements from A and B

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- The symbols of the rational language do not appear in members of two-level sets.
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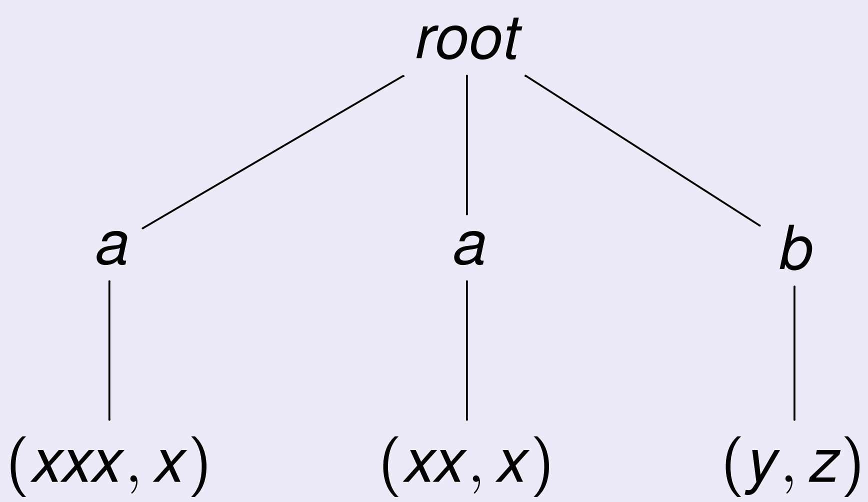

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Remark 2

- Some disjunctions may be expressed at both levels.
- Example:
  $L_1 = (a|b) \;\; \mu_1(a) = (x, x) \;\; \mu_1(b) = (y, y)$
  $L_2 = c \;\; \mu_2(c) = (x, x)|(y, y)$
  $TGS((a|b), \mu_1) = TGS(c, \mu_2) = \{(x, x), (y, y)\}$
- It is not the case of product (and star):
  $(x, x)(y, y) \neq (xy, xy)$

Two-grain Relations

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Proposition

Two-grain sets and two-grain relations are equivalent.

Closure properties

Closed

- union
- product
- star

Not closed

- intersection
  basically because rational relations are not

Sub-classes closed under intersection

Sufficient condition

If the relations in the images of μ_1 and μ_2 belong to a subclass of rational relation closed under intersection, then $TGS(L_1, \mu_1) \cap TGS(L_2, \mu_2)$ is a two-grain set.

Three instances

- recognizable relations
  Cartesian product of independent languages
- length-preserving relations
  Rational language over symbol pairs
- synchronized relations
  product of a length preserving relation and a rational language

Generalization

Three ways

- more than two sizes of grains
- more than two components in the relation (n-tuples instead of pairs)
- possibly different number of grains in the different components

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- three grains: morpheme, grapheme to phoneme matching unit, symbol
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3-tuple

morph. desc.	[type=root][cat=nom]		[type=suffix][cat=adv]	
graphemes	s h	y	l	y
phonemes	$\int$	a ɪ	l	ɪ

Tree structured description

Grains definition

A tree structure defines:

- the number of components (= number of leaves)
- the number of grains (1+ # of inner nodes)
- the grains used by each component (path from the root to the leave)
- the embedding of grains (daughter embedded in the parent)

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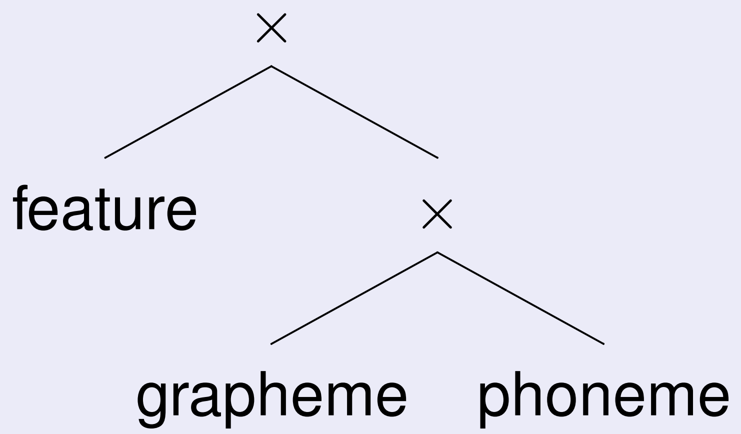
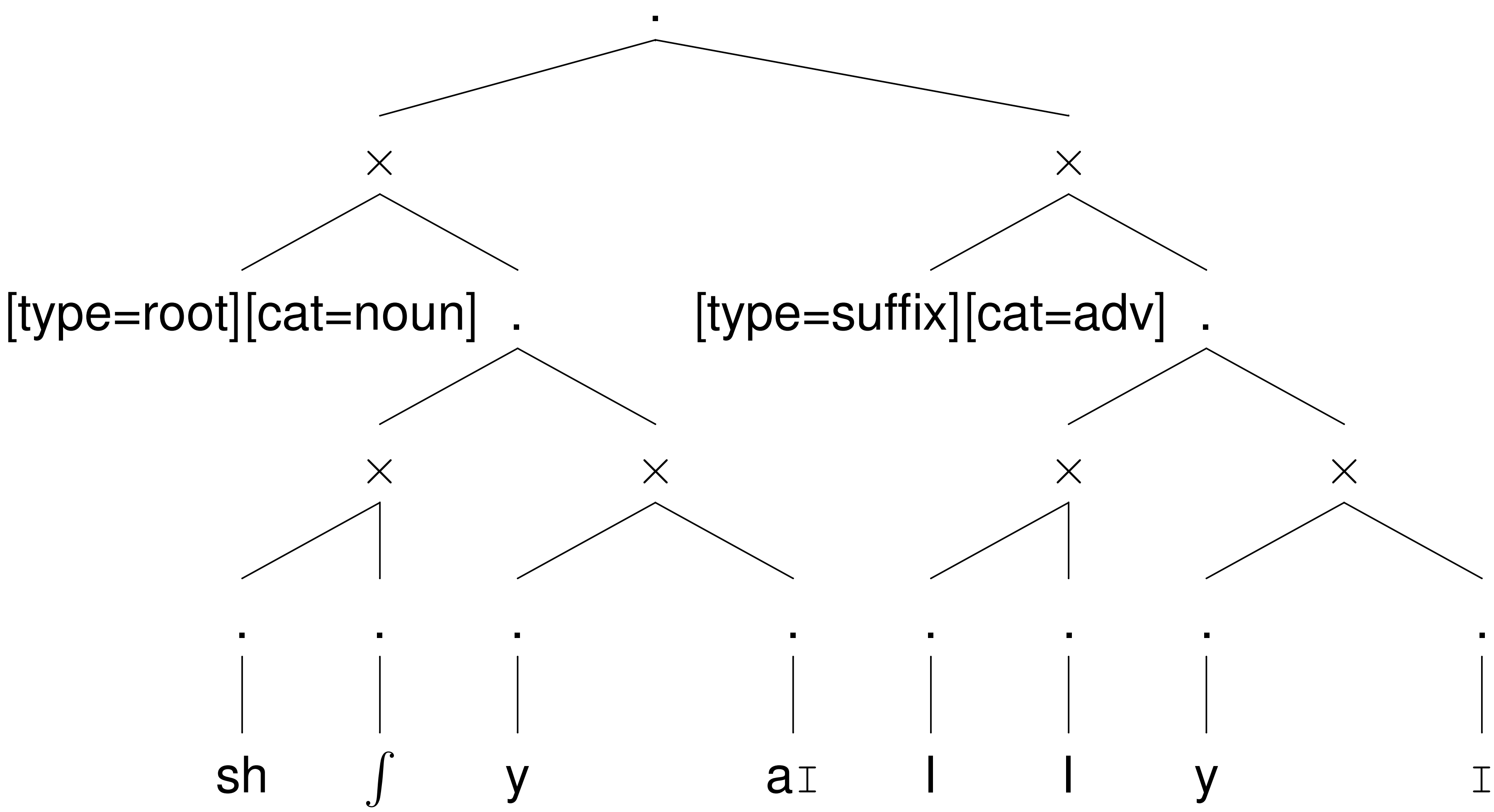


Implicit

Sequences of symbols and grains remain implicit.

The product-Cartesian product structure



Tree linearization

- a special symbol used as terminator for each different grain type.
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[type=root][cat=noun]				ω_2	[type=suffix][cat=noun]				ω_2
s h	ω_1	y	ω_1	ω_2	l	ω_1	y	ω_1	ω_2
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- description of rational sets of tree linearization
- compilable in a finite-state machine

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In the paper, definitions of of tree sets, linearization sets, etc.

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- a finite structure of grains + linear sequences
- remaining finite-state
- closure under intersection possible
- each grain is a level of synchronization between components of the relation
- in practice, more convenient than other subclasses of relation closed under intersection

Future work

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- improving the formalization of the generalization.
- considering parenthesis instead of terminators to denote the structure in strings.
- tree operations implementation within the finite-state framework

Thank you