Syntax-Directed Translations and Quasi-Alphabetic Tree Bimorphisms

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Introduction (I)

- **Finite-state recognizers** and **transducers** have attractive properties and a well-developed theory, and are used with considerable success in natural language processing (e.g., speech and text recognition).
- They cannot perform some of the **syntax-sensitive transformations** and **reorderings** of parts of sentences frequently encountered in translations from one natural language to another.
- researchers switch attention to devices that can model trees and tree transformations:
  - **Tree transducers**;
  - **Synchronous grammars**: Syntax-Directed Translation Schema.
- In a SDTS, parse trees for the input string and the output string are implicitly produced in parallel.

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Introduction (II)

- In natural language processing, **closure under composition** and **preservation of recognizability** are important features of translations [Knight & Graehl 2005, Knight & Hopkins & Graehl 2006].
- The tree transformations defined by many common types of tree transducers are neither closed under composition nor do they preserve the recognizability of tree languages.
- The mathematical framework of synchronous grammars appears less suitable for such operations $\Longrightarrow$ look for alternative formalisms.
- **Tree bimorphisms** provide an elegant algebraic tool for describing various classes of tree transformastions and proving their properties. By taking yields of the input and output trees, tree bimorphisms are transformed into **string-to-string translating devices**.
- How about describing synchronous-rewriting systems with the help of tree bimorphisms? [Shieber 2004]

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Syntax-Directed Translation Schema (I)

What Is an SDTS?

It is a CF grammar with translation elements attached to each production. Whenever a production is used in the derivation of an input sentence, the associated translation element generates a part of the output sentence. [Aho & Ullman 1972].

Definition (Syntax-Directed Translation Schema)

A **SDTS** is a device **T**$= (N, X, Y, P, S)$, where:

- N is a finite set of **nonterminal symbols**,
- X is a finite **input alphabet**,
- Y is a finite **output alphabet**,
- $S \in N$ is the start symbol, and
- P is a finite set of **productions** of the form:

$$p = A \to u_1 A_1 u_2 \ldots u_m A_m u_{m+1}, v_1 A_{\pi(1)} v_2 \ldots v_m A_{\pi(m)} v_{m+1} \quad (*)$$

where $m \geq 0$, $A, A_1, \ldots, A_m \in N$, π is a permutation of $[m]$, and for every $i \in [m+1]$, $u_i = x_1^i \ldots x_{k_i}^i \in X^*$ $(k_i \geq 0)$ and $v_i = y_1^i \ldots y_{l_i}^i \in Y^*$ $(l_i \geq 0)$.

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Syntax-Directed Translation Schema (II)

Example

Let $T = (\{S, A, B\}, \{a, b\}, \{x, y, z\}, P, S)$, where P has the rules:

$$p_1 = S \rightarrow \mathrm{a}\,A\,\mathrm{bb}\,B\,\mathrm{a}\,B\,;\ B\,A\,\mathrm{zyx}\,B\,\mathrm{xx},$$
$$p_2 = A \rightarrow A\,A\,;\ A\,\mathrm{x}\,A,$$
$$p_3 = A \rightarrow \lambda\,;\ \lambda, \text{ and}$$
$$p_4 = B \rightarrow \mathrm{ab}\,;\ \lambda.$$

A **derivation** in T is:

$$(S, S) \overset{p_1}{\Longrightarrow}_T (aAbbBaB, BAzyxBxx) \overset{p_2}{\Longrightarrow}_T (aAAbbBaB, BAxAzyxBxx) \overset{p_3}{\Longrightarrow}{}^*_T$$
$$\overset{p_3}{\Longrightarrow}{}^*_T (abbBaB, BzyxBxx) \overset{p_4}{\Longrightarrow}{}^*_T (abbabaab, zyxxx).$$

Definition (Syntax-Directed Translation)

The **translation** defined by a SDTS T is the relation

$$\tau_T = \{(u, v) \in X^* \times Y^* \mid (S, S) \Rightarrow^*_T (u, v)\},$$

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Local Tree Languages

Let Σ be a ranked alphabet, X a leaf alphabet, and $T_\Sigma(X)$ the set of Σ-terms with variables in X (labeled trees).

What Is a Fork?

The set $\mathrm{fork}(t)$ of **forks** of a ΣX-tree t is defined as:

1. $\mathrm{fork}(d) = \emptyset$ for $d \in X \cup \Sigma_0$;
2. $\mathrm{fork}(t) = \mathrm{fork}(t_1) \cup \ldots \cup \mathrm{fork}(t_m) \cup \{f(\mathrm{root}(t_1), \ldots, \mathrm{root}(t_m))\}$ for $t = f(t_1, \ldots, t_m)$ $(m > 0)$.

The (finite) set of all possible forks of ΣX-trees is denoted by $\mathrm{fork}(\Sigma, X)$.

For any $D \subseteq \Sigma \cup X$ and $E \subseteq \mathrm{fork}(\Sigma, X)$

$$L(D, E) = \{t \in T_\Sigma(X) \mid \mathrm{root}(t) \in D, \mathrm{fork}(t) \subseteq E\}.$$

A ΣX-tree language R is local (in the strict sense), if $R = L(D, E)$ for some D and E.

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Quasi-Alphabetic Tree Bimorphisms (I)

Definition (Tree Homomorphism)

A **tree homomorphism** $\varphi : T_\Sigma(X) \to T_\Omega(Y)$ is determined by a mapping $\varphi_X : X \to T_\Omega(Y)$ and mappings $\varphi_m : \Sigma_m \to T_\Omega(Y \cup \Xi_m)$ $(m \geq 0,\ \Sigma_m \neq \emptyset)$:

1. $x\varphi = \varphi_X(x)$ for any $x \in X$,
2. $c\varphi = \varphi_0(c)$ for any $c \in \Sigma_0$, and
3. $t\varphi = \varphi_m(f)(\xi_1 \leftarrow t_1\varphi, \ldots, \xi_m \leftarrow t_m\varphi)$ for $t = f(t_1, \ldots, t_m)$ $(m > 0)$.

Definition (Quasi-Alphabetic Tree Homomorphism)

We call a tree homomorphism $\varphi : T_\Sigma(X) \to T_\Omega(Y)$ **quasi-alphabetic**, if

1. $\varphi_X(x) \in Y$ for every $x \in X$, and
2. for all $m \geq 0$ and $f \in \Sigma_m$, $\varphi_m(f)$ is of the form

where σ is a permutation of $[m]$, for each $i \in [m+1]$, $k_i \geq 0$ and ... and $g \in \Omega_{m'}$ for $m' = m + k_1 + \cdots + k_{m+1}$.

Let qH denote the class of all quasi-alphabetic tree homomorphisms.

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where σ is a permutation of $[m]$, for each $i \in [m+1]$, $k_i \geq 0$ and $y_i^1, \ldots, y_i^{k_i} \in Y$, and $g \in \Omega_{m'}$ for $m' = m + k_1 + \ldots + k_{m+1}$.

Let qH denote the class of all quasi-alphabetic tree homomorphisms.

Quasi-Alphabetic Tree Bimorphisms (I)

Definition (Tree Homomorphism)

A **tree homomorphism** $\varphi : T_\Sigma(X) \to T_\Omega(Y)$ is determined by a mapping $\varphi_\mathbf{X} : X \to T_\Omega(Y)$ and mappings $\varphi_\mathbf{m} : \Sigma_m \to T_\Omega(Y \cup \Xi_m)$ ($m \geq 0$, $\Sigma_m \neq \emptyset$):

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2. $c\varphi = \varphi_0(c)$ for any $c \in \Sigma_0$, and
3. $t\varphi = \varphi_m(f)(\xi_1 \leftarrow t_1\varphi, \ldots, \xi_m \leftarrow t_m\varphi)$ for $t = f(t_1, \ldots, t_m)$ ($m > 0$).

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We call a tree homomorphism $\varphi : T_\Sigma(X) \to T_\Omega(Y)$ **quasi-alphabetic**, if

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Basic Properties

A quasi-alphabetic tree homomorphism is

- linear: no copying,
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A **tree bimorphism** is a triple $B = (\varphi, R, \psi)$, where

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Notations

For any classes H_1 and H_2 of tree homomorphisms and any class $\mathcal{R}$ of regular tree languages,

- $\mathbf{B}(H_1, \mathcal{R}, H_2)$ are all tree bimorphisms $B = (\varphi, R, \psi)$ with $\varphi \in H_1$, $R \in \mathcal{R}$ and $\psi \in H_2$;
- $\mathcal{B}(H_1, \mathcal{R}, H_2)$ is the corresponding class of tree transformations;
- $\mathbf{B}(\mathrm{qH}, \mathrm{Loc}, \mathrm{qH})$ is the class of **quasi-alphabetic tree bimorphisms** in which the two tree homomorphisms are quasi-alphabetic and the tree language is local;
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An example

Let $B = (\varphi, R, \psi)$ be a bimorphism, where:

- $Z = \{z\}$, $\Gamma_3 = \{f\}$, $\Gamma_0 = \{c\}$, $\Gamma = \Gamma_3 \cup \Gamma_0$;
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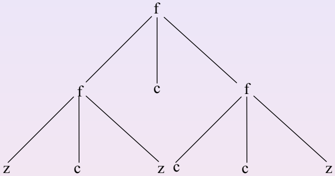

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We applied $\varphi_3(f)=g(y, \xi_2, \xi_1, x, x, \xi_3)$
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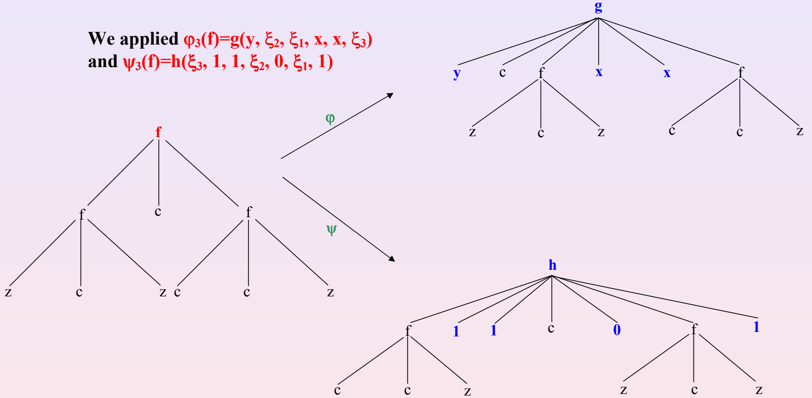

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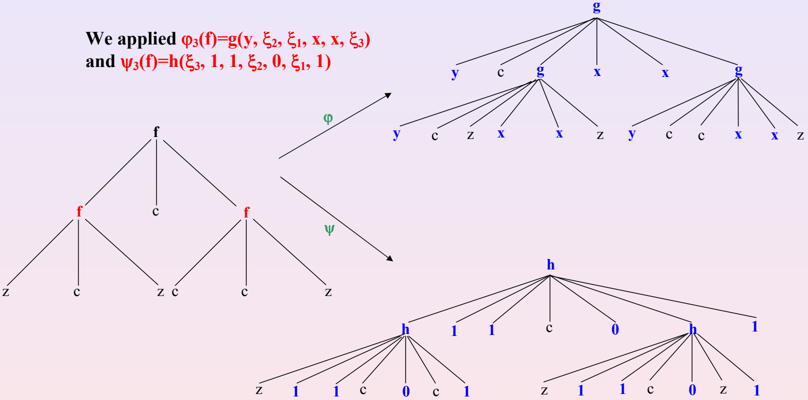

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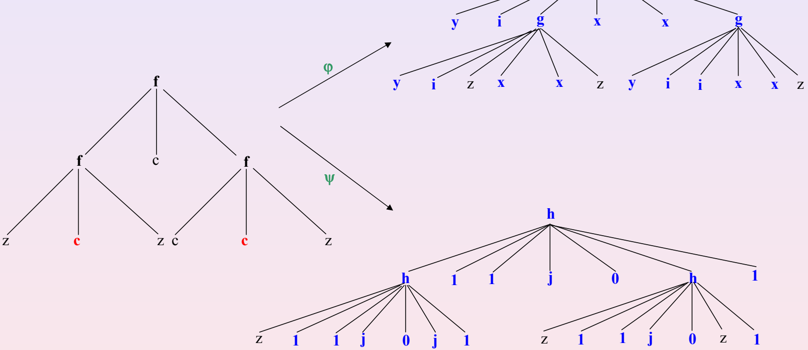

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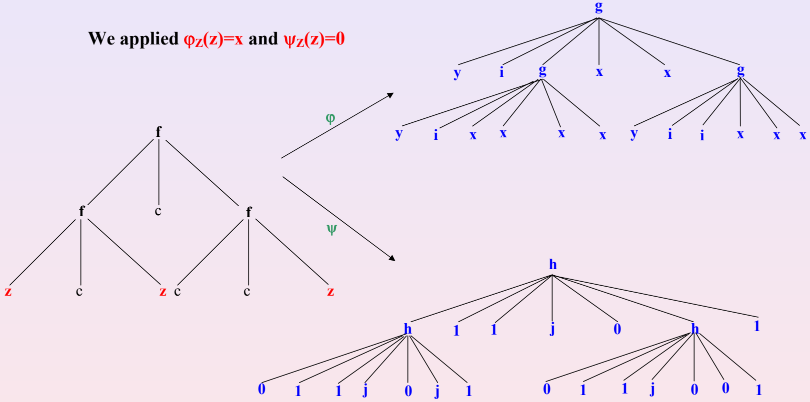
We applied $\varphi_Z(z)=x$ and $\psi_Z(z)=0$



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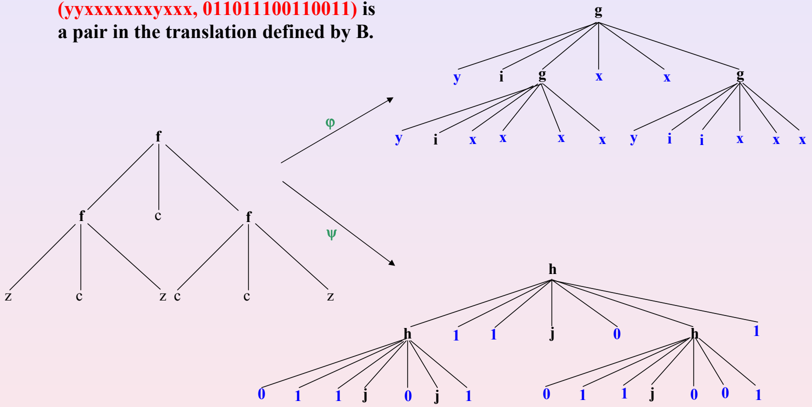
(yyxxxxxxxyxxx, 011011100110011) is a pair in the translation defined by B.



The Connection between SDTSs and $\mathbf{B}(\mathrm{qH}, \mathrm{Loc}, \mathrm{qH})$

Proposition

For every SDTS T, one can define a tree bimorphism B in $\mathbf{B}(\mathrm{qH}, \mathrm{Loc}, \mathrm{qH})$ such that $\tau_T = \mathrm{yd}(\tau_B)$.

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For each quasi-alphabetic tree bimorphism B, one can define a syntax-directed translation schema T such that $\mathrm{yd}(\tau_B) = \tau_T$.

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The class of syntax-directed translations is effectively equal to the class of translations defined by quasi-alphabetic tree bimorphisms.

Lemma (Preservation of Recognizability)

Quasi-alphabetic tree bimorphisms preserve recognizability, i.e., if $B \in \mathbf{B}(\mathrm{qH}, \mathrm{Loc}, \mathrm{qH})$ and R' is a regular tree language, then so is $R'\tau_B$.

Using the connection expressed by the above theorem and the tree language theory, properties of syntax-directed translations can be proved (e.g., the domain and range are context-free languages).

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Classes Equivalent to $\mathbf{B}(\mathrm{qH}, \mathrm{Loc}, \mathrm{qH})$

The essential feature of the bimorphisms in the class $\mathbf{B}(\mathrm{qH}, \mathrm{Loc}, \mathrm{qH})$ is that the tree homomorphisms are quasi-alphabetic; we may either limit or extend the class of tree languages allowed.

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The class of all syntax-directed translations is effectively equal to the class of translations defined by the tree bimorphisms belonging to the class $\mathbf{B}(\mathrm{qH}, \mathrm{Loc} \cap \mathrm{DRec}, \mathrm{qH})$.

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The class $\mathcal{B}(\mathrm{qH}, \mathrm{Loc}, \mathrm{qH})$ is closed under inverses.

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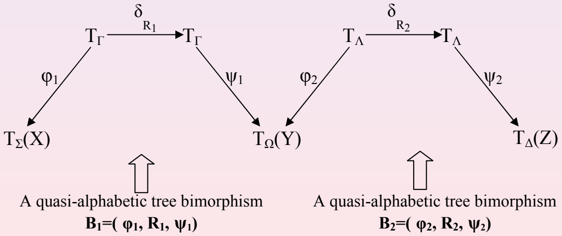

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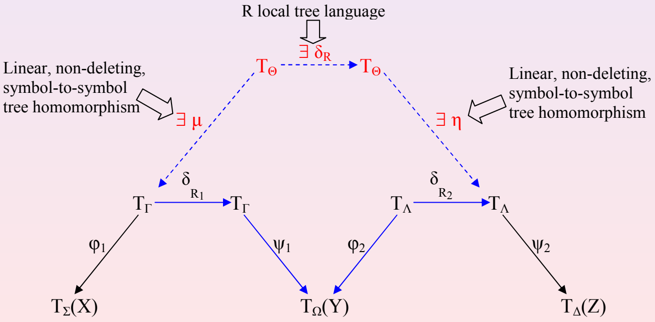

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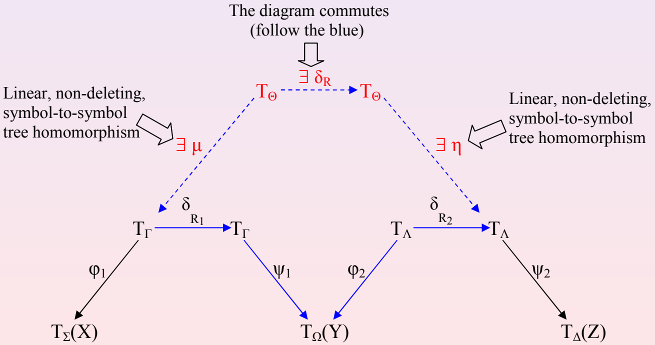

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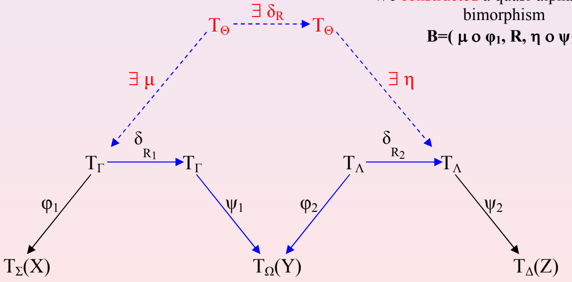

We constructed a quasi-alphabetic tree bimorphism

B=(μ o φ₁, R, η o ψ₁)

Conclusions

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- We introduce the new class of quasi-alphabetic tree bimorphisms.
- We show that the class of syntax-directed translations is effectively equal to the class of translations defined by quasi-alphabetic tree bimorphisms.
- We show that the class of tree transformations defined by quasi-alphabetic tree bimorphisms is closed under composition and inverses, and preserves recognizability $\Longrightarrow$ we solved (partially) an open problem mentioned by Shierber [Shieber 2004], pp.8: "...the bimorphism characterization of tree transducers has led to a series of composition closure results. Similar techniques may now be applicable to synchronous formalisms, where no composition results are known..."

Other (Recent) Work

- There is a **super class** of tree transformations closed under composition [Maletti 2007].
- Synchronous context-free grammars [Satta & Peserico 2005] perform the same translations as quasi-alphabetic tree bimorphisms [Tîrnăucă 2007].

Future Work

- to investigate other closure properties of $\mathbf{B}(\mathrm{qH}, \mathrm{Loc}, \mathrm{qH})$ (e.g., intersection);
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That's all folks!

Thank you!